Math 75A **Maria Nogin**

Practice test 3 - Solutions

Multiple choice questions: circle the correct answer

1. Find the derivative of $\sqrt{2x}$.

A. $\frac{2}{\sqrt{x}}$ **B.** $\frac{2}{\sqrt{2x}}$ **C.** $\frac{1}{2\sqrt{x}}$ (**D.**) $\frac{1}{\sqrt{2x}}$ **E.** $\frac{1}{2\sqrt{2x}}$

2. Evaluate the limit: $\lim_{x\to 0} \frac{\sin(3x)}{5x}$

A. 0 (**B.**) 0.6 **C.** $\frac{1}{5}$ **D.** $\frac{5}{3}$ **E.** Does not exist

3. Simplify the expression: $\frac{8x^3\sqrt{x}}{(3x^2)^2 + 7x^4}$

A. $\frac{8\sqrt{x}}{10x^2}$ **B.** $\frac{\sqrt{x}}{2}$ (**C.**) $\frac{1}{2\sqrt{x}}$ **D.** $\frac{4}{5\sqrt{x}}$ **E.** $4\sqrt{x}$

4. The position of an object at time t is given by $s(t) = 4\sin(t) + 2\cos(t)$. Find the velocity of this object at $t = \frac{\pi}{3}$.

A. $1+\sqrt{3}$ **B.** $1+2\sqrt{3}$ **C.** $1-2\sqrt{3}$ **D.** $2+\sqrt{3}$ (**E.**) $2-\sqrt{3}$

5. Find the equation of the line tangent to the curve $y = x^2 + 4x + 4$ at $(1, 9)$.

A. $y = 9x$ **B.** $y = 6x - 15$ (**C.**) $y = 6x + 3$ **D.** $y = 2x + 1$
E. None of the above

6. If $f(3) = 2$, $f'(3) = 4$, $g(3) = 5$, and $g'(3) = 6$, then the derivative of $\frac{f(x)}{g(x)}$ at $x = 3$ is $\left(\frac{f}{g}\right)'(3) =$

(**A.**) 0.32 **B.** 2/3 **C.** $-8/25$ **D.** 0 **E.** Undefined

7. If $f(x) = 4^{3x}$, find $f'(x)$.

A. 4^{3x} **B.** $3 \cdot 4^{3x}$ **C.** 12^{3x} **D.** $\ln(4)4^{3x}$ (**E.**) $3\ln(4)4^{3x}$

Regular problems: show all your work

8. Differentiate the following functions:

(a) $f(x) = 7x - 3$
$f'(x) = 7$

(b) $p(s) = s^5 - 2s^4 + 3s^3 - 4s^2 + 5s - 6$
$p'(s) = 5s^4 - 8s^3 + 9s^2 - 8s + 5$

(c) $f(t) = \dfrac{3t^2 - 5t + 1}{\sqrt{t}}$

$f(t) = 3t^{1.5} - 5t^{0.5} + t^{-0.5}$

$f'(t) = 4.5t^{0.5} - 2.5t^{-0.5} - 0.5t^{-1.5} = 4.5\sqrt{t} - \dfrac{2.5}{\sqrt{t}} - \dfrac{1}{2t^{1.5}}$

(d) $g(x) = x^2 - \dfrac{x^3}{\sqrt[4]{x}} + \dfrac{3}{x}$

$g(x) = x^2 - x^{11/4} + 3x^{-1}$

$g'(x) = 2x - \dfrac{11}{4}x^{7/4} - 3x^{-2}$

(e) $q(y) = \dfrac{y^2 + y + 1}{y + 1}$

$q'(y) = \dfrac{(2y+1)(y+1) - (y^2+y+1)(1)}{(y+1)^2} = \dfrac{y^2 + 2y}{(y+1)^2}$

(f) $y = 3\sin(x^5) + \dfrac{\pi}{2}$

$y' = 3\cos(x^5) \cdot 5x^4 = 15x^4\cos(x^5)$

(g) $f(x) = \cos(4)(x^3 - 3x)$

$f'(x) = \cos(4)(3x^2 - 3)$

(h) $g(x) = \dfrac{x^3 - 5}{\cos(-x)}$

$g'(x) = \dfrac{3x^2\cos x + (x^3 - 5)\sin x}{\cos^2 x}$

(i) $h(x) = \tan(x)\left(\dfrac{1}{\sqrt[4]{x^3}} + \dfrac{2}{x}\right)$

$h'(x) = \sec^2(x)\left(\dfrac{1}{\sqrt[4]{x^3}} + \dfrac{2}{x}\right) + \tan(x)\left(-\dfrac{3}{4}x^{-\frac{7}{4}} - \dfrac{2}{x^2}\right)$

(j) $f(t) = 5e^x - 8 \cdot 3^x + 9x^2$

$f'(t) = 5e^x - 8\ln(3)3^x + 18x$

9. Find the points where the tangent line to the graph of $f(x) = x^5 - 80x$ is horizontal.

The tangent line is horizontal when $f'(x) = 0$.

$f'(x) = 5x^4 - 80 = 0$

$5(x^4 - 16) = 0$

$5(x^2 - 4)(x^2 + 4) = 0$

$5(x-2)(x+2)(x^2+4) = 0$

$x = 2$ and $x = -2$

Thus the tangent line is horizontal at $(2, -128)$ and $(-2, 128)$.

10. Find an equation of the tangent line to $y = \sqrt{2x+3}$ at $(3, 3)$.

The slope of the tangent line is equal to the derivative at 3.

$y' = \dfrac{1}{2\sqrt{2x+3}} \cdot 2 = \dfrac{1}{\sqrt{2x+3}}$.

$y'(3) = \dfrac{1}{3}$

$y - 3 = \dfrac{1}{3}(x - 3)$

$y = \dfrac{1}{3}x + 2.$

11. Evaluate the limits:

(a) $\lim_{x\to 0}\frac{\sin(6x)}{\sin(7x)} = \lim_{x\to 0}\frac{\frac{\sin(6x)}{x}}{\frac{\sin(7x)}{x}} = \lim_{x\to 0}\frac{\frac{\sin(6x)}{\frac{1}{6}\cdot 6x}}{\frac{\sin(7x)}{\frac{1}{7}\cdot 7x}} = \frac{\lim_{x\to 0}\frac{\sin(6x)}{\frac{1}{6}\cdot 6x}}{\lim_{x\to 0}\frac{\sin(7x)}{\frac{1}{7}\cdot 7x}} = \frac{\frac{1}{1/6}}{\frac{1}{1/7}} = \frac{6}{7}$

(b) $\lim_{x\to 0}\frac{2x}{\tan(4x)} = \lim_{x\to 0}\frac{2x\cos(4x)}{\sin(4x)} = \lim_{x\to 0}\frac{2\cos(4x)}{\frac{\sin(4x)}{x}} = \lim_{x\to 0}\frac{2\cos(4x)}{\frac{\sin(4x)}{\frac{1}{4}\cdot 4x}} = \frac{\lim_{x\to 0} 2\cos(4x)}{\lim_{x\to 0}\frac{\sin(4x)}{\frac{1}{4}\cdot 4x}} =$
$\frac{2}{\frac{1}{1/4}} = \frac{2}{4} = \frac{1}{2}$

(c) $\lim_{x\to 0}\tan(5x)\csc(x) = \lim_{x\to 0}\frac{\sin(5x)}{\cos(5x)\sin(x)} = \lim_{x\to 0}\frac{\frac{\sin(5x)}{x}}{\cos(5x)\frac{\sin(x)}{x}} = \lim_{x\to 0}\frac{\frac{\sin(5x)}{\frac{1}{5}\cdot 5x}}{\cos(5x)\frac{\sin(x)}{x}} =$
$\frac{\lim_{x\to 0}\frac{\sin(5x)}{\frac{1}{5}\cdot 5x}}{\lim_{x\to 0}\cos(5x)\frac{\sin(x)}{x}} = \frac{\frac{1}{1/5}}{1} = 5$

12. Solve for $\frac{dy}{dx}$: $5x\left(8y\frac{dy}{dx} + x^2\right) = 7\frac{dy}{dx} - 3xy^3$.

$40xy\frac{dy}{dx} + 5x^3 = 7\frac{dy}{dx} - 3xy^3$

$40xy\frac{dy}{dx} - 7\frac{dy}{dx} = -3xy^3 - 5x^3$

$(40xy - 7)\frac{dy}{dx} = -3xy^3 - 5x^3$

$\frac{dy}{dx} = \frac{-3xy^3 - 5x^3}{40xy - 7} = \frac{3xy^3 + 5x^3}{7 - 40xy}$

13. Consider the curve given by $x^3y^3 - 3xy^3 + 4y = 6$.

(a) Use implicit differentiation to find $y'(x)$.
$3x^2y^3 + x^3 3y^2y' - 3y^3 - 3x3y^2y' + 4y' = 0$
$3x^3y^2y' - 9xy^2y' + 4y' = 3y^3 - 3x^2y^3$
$(3x^3y^2 - 9xy^2 + 4)y' = 3y^3 - 3x^2y^3$
$y' = \frac{3y^3 - 3x^2y^3}{3x^3y^2 - 9xy^2 + 4}$

(b) Check that the point $(2, 1)$ lies on this curve.
$2^3 \cdot 1^3 - 3 \cdot 2 \cdot 1^3 + 4 \cdot 1 = 6.$

(c) What is the slope of the tangent line to this curve at $(2, 1)$?
$y'(2) = \frac{3 \cdot 1^3 - 3 \cdot 2^2 \cdot 1^3}{3 \cdot 2^3 \cdot 1^2 - 9 \cdot 2 \cdot 1^2 + 4} = -0.9.$

14. A snowball is melting (so it is decreasing). Find the rate of decrease of its volume with respect to the radius when the radius is 3 cm. (Recall that the volume of a ball is $V = \frac{4}{3}\pi r^3$.)

$V(r) = \frac{4}{3}\pi r^3$

$V'(r) = 4\pi r^2$

If $r = 3$, $V' = 4\pi \cdot 9 = 36\pi$.

So the rate of decrease of the volume with respect to the radius is 36π.