Review - 3

THEORY

Inverse function

A function $f : A \to B$ is called one-to-one if it never takes on the same value twice, i.e. if for any $x_1 \neq x_2$ in A, $f(x_1) \neq f(x_2)$ in B.

If $f : A \to B$ is a one-to-one function, then its inverse function is $f^{-1} : B \to A$ defined by $f^{-1}(x) = y$ if $f(y) = x$.

Cancellation laws: $f(f^{-1}(x)) = x$, $f^{-1}(f(x)) = x$.

The graph of the inverse function is obtained by reflecting the graph of the original function about the line $y = x$.

Example: the exponential and logarithmic functions with the same base a are inverse functions of each other.

To find the inverse function of $f(x)$:

1. Write $y = f(x)$.
2. Solve $y = f(x)$ for x. You'll get the equation $x = f^{-1}(y)$.
3. Switch x and y in the obtained equation to get $y = f^{-1}(x)$.

Exponential and logarithmic functions: definitions

An exponential function is a function of the form $f(x) = a^x$ (where $a > 0$).

A logarithmic function is a function of the form $f(x) = \log_a x$ (where $a > 0$, $a \neq 0$) defined by

$$\log_a x = y \quad \text{if} \quad a^y = x.$$

For $a = e \approx 2.7$: $f(x) = e^x$ is called the natural exponential function;
$f(x) = \log_a x = \ln x$ is called the natural logarithmic function.

Graphs of exponential and logarithmic functions

See sections 3.1 and 3.2.

Derivatives of exponential and logarithmic functions

- Exponential functions: $(e^x)' = e^x$, $(a^x)' = \ln(a)a^x$
- Logarithmic functions: $(\ln x)' = \dfrac{1}{x}$, $(\log_a x)' = \dfrac{1}{x \ln a}$

Laws of exponents

- $a^b \cdot a^c = a^{b+c}$
- $\dfrac{a^b}{a^c} = a^{b-c}$
- $\left(a^b\right)^c = \left(a^c\right)^b = a^{bc}$
- $\sqrt[b]{a} = a^{1/b}$
- $\dfrac{1}{a^b} = a^{-b}$
- $a^0 = 1$
- $a^1 = a$

Laws of logarithms

- $\log_a(bc) = \log_a b + \log_a c$
- $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$
- $\log_a(b^c) = c \log_a b$
- $\log_a b = \dfrac{\log_c b}{\log_c a}$
- $\log_a 1 = 0$
- $\log_a a = 1$

Cancellation laws

- $\log_a(a^x) = x$
- $a^{\log_a x} = x$
- $\ln(e^x) = x$
- $e^{\ln x} = x$