Math 75A **Maria Nogin**

Practice test 2 - Solutions

Multiple choice questions: circle the correct answer

1. How many vertical asymptotes does the curve $y = \dfrac{x+1}{x(x+2)(x+3)}$ have?

   A. 0 **B.** 1 **C.** 2 (**D.**) 3 **E.** 4

   ($x = 0$, $x = -2$, and $x = -3$)

2. $\displaystyle\lim_{x\to 2} \frac{5}{x-2} =$

   A. 0 **B.** 5 **C.** ∞ **D.** $-\infty$ (**E.**) Does not exist

   (because the limits from the right and from the left are not equal)

3. $\displaystyle\lim_{x\to -\infty} \frac{x+2}{3x+4} =$

   A. 1 **B.** $\frac{1}{2}$ (**C.**) $\frac{1}{3}$ **D.** 0 **E.** Does not exist

   (divide the numerator and denominator by x)

4. Find the rate of change of $y = 3x + 5$ at $x = 4$.

   (**A.**) 3 **B.** 4 **C.** 5 **D.** 17
   E. None of the above

   (the rate of change is $y'(4)$, the slope of the graph, which is 3)

5. Find the derivative of $\sqrt{2x}$.

   A. $\dfrac{2}{\sqrt{x}}$ **B.** $\dfrac{2}{\sqrt{2x}}$ **C.** $\dfrac{1}{2\sqrt{x}}$ (**D.**) $\dfrac{1}{\sqrt{2x}}$ **E.** $\dfrac{1}{2\sqrt{2x}}$

   (either rewrite the function as $\sqrt{2}\sqrt{x}$ and then use the constant multiple rule, or use the chain rule; simplify your answer)

6. Simplify the expression: $\dfrac{8x^3\sqrt{x}}{(3x^2)^2 + 7x^4}$

   A. $\dfrac{8\sqrt{x}}{10x^2}$ **B.** $\dfrac{\sqrt{x}}{2}$ (**C.**) $\dfrac{1}{2\sqrt{x}}$ **D.** $\dfrac{4}{5\sqrt{x}}$ **E.** $4\sqrt{x}$

   (use the rules of exponents)

7. The position of an object at time t is given by $s(t) = 4\sin(t) + 2\cos(t)$. Find the velocity of this object at $t = \dfrac{\pi}{3}$.

   A. $1+\sqrt{3}$ **B.** $1+2\sqrt{3}$ **C.** $1-2\sqrt{3}$ **D.** $2+\sqrt{3}$ (**E.**) $2-\sqrt{3}$

   (the velocity is the derivative of the position function)

8. Find the equation of the line tangent to the curve $y = x^2 + 4x + 4$ at $(1, 9)$.

A. $y = 9x$ **B.** $y = 6x - 15$ **C.** $y = 6x + 3$ **D.** $y = 2x + 1$
E. None of the above

(first find the slope, i.e. $y'(1)$; then use the point-slope equation of the line and simplify)

Regular problems: show all your work

9. Evaluate the limits:

(a) $\lim_{x\to 2^+} \dfrac{x^3-2}{x^2-x-2} = \lim_{x\to 2^+} \dfrac{x^3-2}{(x-2)(x+1)} \left[\dfrac{\text{pos.}}{\text{(small pos.)(pos.)}}\right] = +\infty$

(b) $\lim_{x\to 2^-} \dfrac{x^3-2}{x^2-x-2} = \lim_{x\to 2^+} \dfrac{x^3-2}{(x-2)(x+1)} \left[\dfrac{\text{pos.}}{\text{(small neg.)(pos.)}}\right] = -\infty$

(c) $\lim_{x\to 2} \dfrac{x^3-2}{x^2-x-2}$ DNE because the limits in (d) and (e) are not equal

(d) $\lim_{x\to\infty} \dfrac{5x^3-x-3}{4x^3+3x^2-3} = \lim_{x\to\infty} \dfrac{5-\frac{1}{x^2}-\frac{3}{x^3}}{4+\frac{3}{x}-\frac{3}{x^3}} = \dfrac{5}{4}$

(e) $\lim_{x\to-\infty} \dfrac{5x^2-x-3}{4x^3+3x^2-3} = \lim_{x\to\infty} \dfrac{\frac{5}{x}-\frac{1}{x^2}-\frac{3}{x^3}}{4+\frac{3}{x}-\frac{3}{x^3}} = \dfrac{0}{4} = 0$

(f) $\lim_{x\to\infty} \dfrac{5x^3-x-3}{4x^2+3x-3} = \lim_{x\to\infty} \dfrac{5x-\frac{1}{x}-\frac{3}{x^2}}{4+\frac{3}{x}-\frac{3}{x^2}} = \infty$

(g) $\lim_{x\to\infty} \dfrac{\sqrt{4x^2+5}}{3x-3} = \lim_{x\to\infty} \dfrac{\frac{\sqrt{4x^2+5}}{x}}{3-\frac{3}{x}} = \lim_{x\to\infty} \dfrac{\frac{\sqrt{4x^2+5}}{\sqrt{x^2}}}{3-\frac{3}{x}} = \lim_{x\to\infty} \dfrac{\sqrt{\frac{4x^2+5}{x^2}}}{3-\frac{3}{x}} = \lim_{x\to\infty} \dfrac{\sqrt{4+\frac{5}{x^2}}}{3-\frac{3}{x}} = \dfrac{2}{3}$

(h) $\lim_{x\to-\infty} \dfrac{\sqrt{4x^2+5}}{3x-3} = \lim_{x\to-\infty} \dfrac{\frac{\sqrt{4x^2+5}}{x}}{3-\frac{3}{x}} = \lim_{x\to-\infty} \dfrac{\frac{\sqrt{4x^2+5}}{-\sqrt{x^2}}}{3-\frac{3}{x}} = \lim_{x\to-\infty} \dfrac{-\sqrt{\frac{4x^2+5}{x^2}}}{3-\frac{3}{x}} =$

$\lim_{x\to-\infty} \dfrac{-\sqrt{4+\frac{5}{x^2}}}{3-\frac{3}{x}} = -\dfrac{2}{3}$

(i) $\lim_{x\to\infty} (3-x+2x^2-5x^3) = \lim_{x\to\infty} x^3\left(\dfrac{3}{x^3}-\dfrac{1}{x^2}+\dfrac{2}{x}-5\right) = -\infty$

(j) $\lim_{x\to-\infty} (3-x+2x^2-5x^3) = \lim_{x\to-\infty} x^3\left(\dfrac{3}{x^3}-\dfrac{1}{x^2}+\dfrac{2}{x}-5\right) = \infty$

10. Find the vertical and horizontal asymptotes of $f(x) = \dfrac{(x+2)(3x-4)}{(x-5)(x+7)}$.

Since rational functions are continuous in their domains, $f(x)$ can have vertical asymptotes only at 5 and -7 (where it is undefined). Check the limits of $f(x)$ as x approaches 5 and -7:

$$\lim_{x\to 5^+} \frac{(x+2)(3x-4)}{(x-5)(x+7)} \left[\frac{\text{(pos.)(pos.)}}{\text{(small pos.)(pos.)}}\right] = +\infty$$

$$\lim_{x\to -7^+} \frac{(x+2)(3x-4)}{(x-5)(x+7)} \left[\frac{\text{(neg.)(neg.)}}{\text{(neg.)(small pos.)}}\right] = -\infty$$

Since the limits are infinite, $f(x)$ has vertical asymptotes $x = 5$ and $x = -7$.

To find the horizontal asymptotes, we find the limits at infinity and negative infinity:

$$\lim_{x\to\infty} \frac{(x+2)(3x-4)}{(x-5)(x+7)} = \lim_{x\to\infty} \frac{\frac{(x+2)}{x}\frac{(3x-4)}{x}}{\frac{(x-5)}{x}\frac{(x+7)}{x}} = \lim_{x\to\infty} \frac{(1+\frac{2}{x})(3-\frac{4}{x})}{(1-\frac{5}{x})(1+\frac{7}{x})} = \frac{3}{1} = 3$$

$$\lim_{x\to-\infty} \frac{(x+2)(3x-4)}{(x-5)(x+7)} = \lim_{x\to-\infty} \frac{\frac{(x+2)}{x}\frac{(3x-4)}{x}}{\frac{(x-5)}{x}\frac{(x+7)}{x}} = \lim_{x\to-\infty} \frac{(1+\frac{2}{x})(3-\frac{4}{x})}{(1-\frac{5}{x})(1+\frac{7}{x})} = \frac{3}{1} = 3$$

Thus $y = 3$ is the only horizontal asymptotes.

11. Differentiate the following functions:

(a) $f(x) = 5$
$f'(x) = 0$

(b) $f(x) = 7x - 3$
$f'(x) = 7$

(c) $p(s) = s^5 - 2s^4 + 3s^3 - 4s^2 + 5s - 6$
$p'(s) = 5s^4 - 8s^3 + 9s^2 - 8s + 5$

(d) $f(t) = \sqrt{t}$
$f'(t) = \dfrac{1}{2\sqrt{t}}$

(e) $f(x) = \dfrac{2}{x}$
$f'(x) = -\dfrac{2}{x^2}$

(f) $f(t) = \dfrac{3t^2 - 5t + 1}{\sqrt{t}}$
$f(t) = 3t^{1.5} - 5t^{0.5} + t^{-0.5}$
$f'(t) = 4.5t^{0.5} - 2.5t^{-0.5} - 0.5t^{-1.5} = 4.5\sqrt{t} - \dfrac{2.5}{\sqrt{t}} - \dfrac{1}{2t^{1.5}}$

(g) $g(x) = x^2 - \dfrac{x^3}{\sqrt[4]{x}} + \dfrac{3}{x}$
$g(x) = x^2 - x^{11/4} + 3x^{-1}$
$g'(x) = 2x - \dfrac{11}{4}x^{7/4} - 3x^{-2}$

(h) $q(y) = \dfrac{y^2 + y + 1}{y + 1}$
$q'(y) = \dfrac{(2y+1)(y+1) - (y^2+y+1)(1)}{(y+1)^2} = \dfrac{y^2 + 2y}{(y+1)^2}$

(i) $y = 3\sin(x^5) + \dfrac{\pi}{2}$
$y' = 3\cos(x^5) \cdot 5x^4 = 15x^4\cos(x^5)$

(j) $f(x) = \cos(4)(x^3 - 3x)$
$f'(x) = \cos(4)(3x^2 - 3)$

(k) $g(x) = \dfrac{x^3 - 5}{\cos(-x)}$

$$g'(x) = \frac{3x^2 \cos x + (x^3 - 5)\sin x}{\cos^2 x}$$

(l) $h(x) = \tan(x)\left(\dfrac{1}{\sqrt[4]{x^3}} + \dfrac{2}{x}\right)$

$$h'(x) = \sec^2(x)\left(\frac{1}{\sqrt[4]{x^3}} + \frac{2}{x}\right) + \tan(x)\left(-\frac{3}{4}x^{-\frac{7}{4}} - \frac{2}{x^2}\right)$$

12. Find the points where the tangent line to the graph of $f(x) = x^5 - 80x$ is horizontal.

    The tangent line is horizontal when $f'(x) = 0$.

    $f'(x) = 5x^4 - 80 = 0$

    $5(x^4 - 16) = 0$

    $5(x^2 - 4)(x^2 + 4) = 0$

    $5(x - 2)(x + 2)(x^2 + 4) = 0$

    $x = 2$ and $x = -2$

    Thus the tangent line is horizontal at $(2, -128)$ and $(-2, 128)$.

13. Find an equation of the tangent line to $y = \sqrt{2x + 3}$ at $(3, 3)$.

    The slope of the tangent line is equal to the derivative at 3.

    $y' = \dfrac{1}{2\sqrt{2x+3}} \cdot 2 = \dfrac{1}{\sqrt{2x+3}}.$

    $y'(3) = \dfrac{1}{3}$

    $y - 3 = \dfrac{1}{3}(x - 3)$

    $y = \dfrac{1}{3}x + 2.$