Math 75A

Final Exam

May 20, 2009

- No calculators, notes, or books are allowed.

- Please turn off your cell phones.

- You have 2 hours to complete the exam.

- The final exam counts as 20 percent of your grade.

- You may write on this test. When you are done please turn in your test and scantron form. The test and answers will be posted on the course web page by Friday.

1. Find an equation of the line that passes through the point $(2, 3)$ and is parallel to the line $3x + y = 5$.

   (a) $y = -3x + 9$

   (b) $y = 3x - 3$

   (c) $y = -3x + 5$

   (d) $y = 3x - 5$

   (e) $y = -\frac{1}{3}x + \frac{11}{3}$

2. Let $f(x) = \begin{cases} -x + 3 & \text{if} \quad x < -5 \\ 4 & \text{if} \quad x = -5 \\ x^2 - 1 & \text{if} \quad -5 < x \leq 5 \\ 2x - 3 & \text{if} \quad x > 5 \end{cases}$

   Find $f(-4)$.

   (a) -17

   (b) -11

   (c) 4

   (d) 7

   (e) 15

3. Find the domain of the function $f(x) = \sqrt{2x + 6}$.

   (a) $[0, \infty)$

   (b) $[3, \infty)$

   (c) $[-3, \infty)$

   (d) $(-3, 3)$

   (e) $(-\infty, -3) \cup (3, \infty)$

4. Let $f = x^2 - 3$ and $g(x) = \sqrt{x} + 2$. Find $(f \circ g)(x)$.

    (a) $x + 1$

    (b) $(x^2 - 3)(\sqrt{x} + 2)$

    (c) $x + 1 + 4\sqrt{x}$

    (d) $\sqrt{x^2 - 3} + 2$

    (e) None of the above

5. Simplify the expression: $(1 - \cos^2 x)(\cot x \cos x + \sin x)$

    (a) 1

    (b) $\sin x$

    (c) $\cos x$

    (d) $\sin^2 x$

    (e) $\cos^2 x$

6. Simplify the expression: $\dfrac{\frac{x+2}{x+1} - \frac{x-1}{x}}{\frac{3}{x} - \frac{1}{x-1}}$

    (a) $-\dfrac{1}{3}$

    (b) $\dfrac{x + 1}{x - 3}$

    (c) $\dfrac{2x - 1}{2x - 1}$

    (d) $\dfrac{2x + 1}{2x - 3}$

    (e) None of the above

7. Let $f(x) = \begin{cases} -x + 3 & \text{if} & x < -5 \\ 4 & \text{if} & x = -5 \\ x^2 - 1 & \text{if} & -5 < x < 5 \\ 2x - 3 & \text{if} & x \geq 5 \end{cases}$

   Find $\lim\limits_{x \to 5^-} f(x)$.

   (a) 0

   (b) 7

   (c) 8

   (d) 24

   (e) Does not exist

8. Evaluate the limit: $\lim\limits_{s \to 1} \dfrac{s^3 - 1}{s^2 - 1}$.

   (a) 0

   (b) $\frac{1}{2}$

   (c) 1

   (d) $\frac{3}{2}$

   (e) Does not exists

9. Find the value of c for which the function

   $$f(x) = \begin{cases} 4x + c & \text{if} & x \in [-\infty, 2] \\ 3cx - 2 & \text{if} & x \in (2, \infty) \end{cases}$$

   is continuous everywhere.

   (a) -2

   (b) -1

   (c) 0

   (d) 1

   (e) 2

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10. Find the vertical asymptote(s) of $f(x) = \dfrac{(x-2)^2}{x^2-4}$.

   (a) $x = 2$

   (b) $x = -2$

   (c) $y = 1$

   (d) $x = -2$ and $x = 2$

   (e) $y = -2$ and $y = 2$

11. Find the horizontal asymptote(s) of $f(x) = \dfrac{5x^2+2}{4x^3-4}$.

   (a) $x = 0$

   (b) $x = 1$

   (c) $y = 0$

   (d) $y = \frac{5}{4}$

   (e) $y = \frac{5}{4}$ and $y = -\frac{5}{4}$

12. Find an equation of the tangent line to the parabola $y = x^2 + 3x + 5$ at the point $(-2, 3)$.

   (a) $y = x - 1$

   (b) $y = x + 1$

   (c) $y = -x - 1$

   (d) $y = -x + 1$

   (e) None of the above

13. A particle moves along a straight line with equation of motion $s(t) = 2\sqrt{t+3}$. Find its average velocity over interval from $t = 1$ to $t = 6$.

   (a) $\frac{2}{5}$

   (b) $\frac{1}{2}$

   (c) $\frac{2}{3}$

   (d) 2

   (e) $\frac{4}{3}$

14. Simplify: $\dfrac{x^7(3x)^3 - 7x^{10}}{5x^3}$

   (a) $4x^7$

   (b) $16x^{\frac{10}{3}}$

   (c) $-\frac{4}{5}x^7$

   (d) $-16x^8$

   (e) None of the above

15. If $f(x) = 6x^2\sqrt{x} + \dfrac{16}{x\sqrt{x}}$, find $f'(4)$.

   (a) 76

   (b) 108

   (c) 119.25

   (d) 120.75

   (e) 194

16. Let $f(x) = \dfrac{e^x}{x+2}$. Find $f'(0)$.

    (a) 0

    (b) $\frac{1}{4}$

    (c) $\frac{1}{2}$

    (d) 1

    (e) Does not exists

17. The values of f, f', g, and g' are as follows.

x	0	1	2	3
$f(x)$	3	2	-1	2
$f'(x)$	4	3	0	-1
$g(x)$	1	2	3	1
$g'(x)$	-1	0	-1	-2

    If $h = f \circ g$, find $h'(0)$.

    (a) 0

    (b) 2

    (c) -8

    (d) -4

    (e) -3

18. Find the derivative of $\cos(2x)$ at $x = \dfrac{\pi}{3}$.

    (a) $-\sqrt{3}$

    (b) $\sqrt{3}$

    (c) $-\dfrac{\sqrt{6}}{2}$

    (d) $\frac{\sqrt{6}}{2}$

    (e) None of the above

19. Which of the following functions is one-to-one?

    (a) $f(x) = 1$

    (b) $f(x) = x^2$

    (c) $f(x) = x^3$

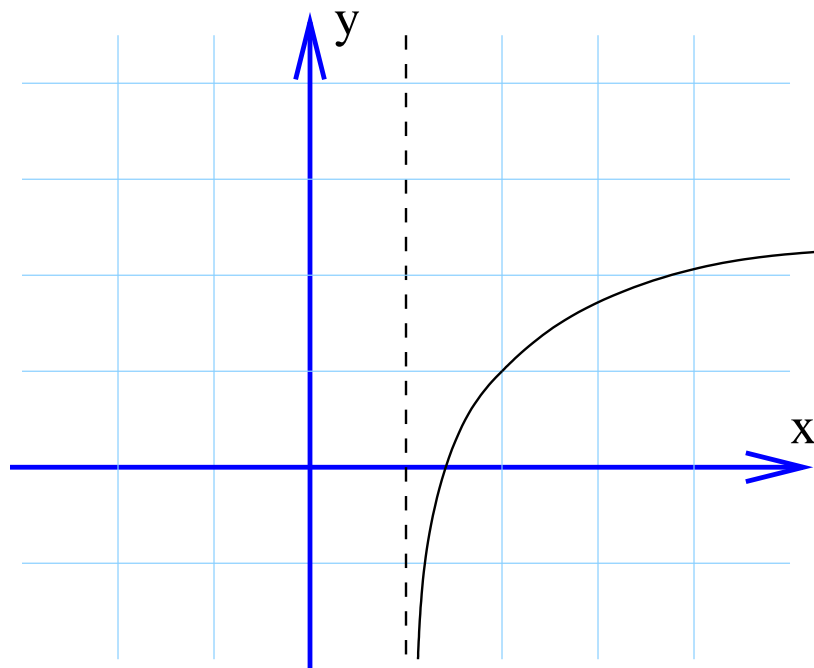   (d) $f(x) = \frac{1}{x^2}$

    (e) $f(x) = \sin x$

20. Find the inverse function of $f(x) = \dfrac{x}{x-3}$

    (a) $f^{-1}(x) = -\dfrac{x}{x-3}$

    (b) $f^{-1}(x) = \dfrac{x-3}{x}$

    (c) $f^{-1}(x) = \dfrac{x}{x+3}$

    (d) $f^{-1}(x) = \dfrac{3x}{x-1}$

    (e) None of the above

21. Evaluate the limit: $\lim\limits_{x \to 0} e^x$.

    (a) 0

    (b) 1

    (c) $-\infty$

    (d) ∞

    (e) Does not exist

22. Which of the following is an equation of the given curve?

(a) $y = e^{x-1} + 1$

(b) $y = \ln(x - 1) + 1$

(c) $y = e^{x+1} - 1$

(d) $y = \ln(x + 1) - 1$

(e) $y = \ln(x + 1) + 1$

23. Evaluate: $2^{2\log_2(2+2^2)}$

(a) 12

(b) 32

(c) 36

(d) 64

(e) 256

24. Solve the equation: $4 - \ln(2 - x) = 0$.

    (a) $\frac{e}{4}$

    (b) $\frac{e^4}{2}$

    (c) $2 - e^4$

    (d) $2e^{-4}$

    (e) None of the above

25. Differentiate: $f(x) = x \log_5 x$.
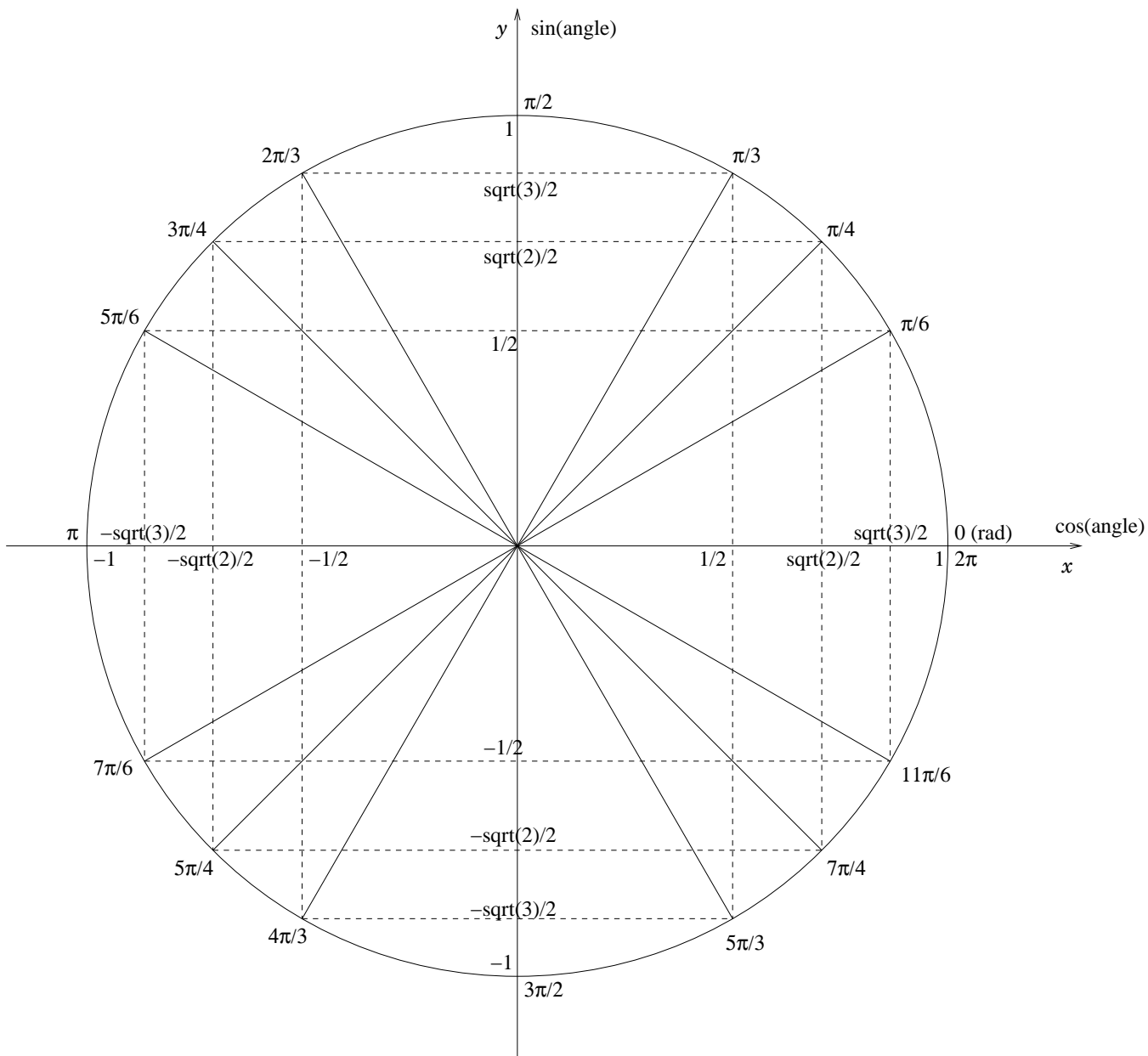
    (a) $f'(x) = \frac{1}{5}$

    (b) $f'(x) = \frac{1}{5x}$

    (c) $f'(x) = \log_5 x + \frac{1}{5}$

    (d) $f'(x) = \log_5 x + x5^x \ln 5$

    (e) None of the above

Please double check all your answers before turning them in. Do not leave any questions unanswered – there is no penalty for guessing!

Good luck on your other final exams, and have a great summer break!

Answer Key:
1. (a)   2. (e)   3. (c)   4. (c)   5. (b)   6. (d)   7. (d)   8. (d)   9. (e)
10. (b)   11. (c)   12. (d)   13. (a)   14. (a)   15. (c)   16. (b)   17. (e)
18. (a)   19. (c)   20. (d)   21. (b)   22. (b)   23. (c)   24. (c)   25. (e)

$$\cos(\alpha) = x \qquad\qquad \sec(\alpha) = \frac{1}{\cos(\alpha)} = \frac{1}{x}$$

$$\sin(\alpha) = y \qquad\qquad \csc(\alpha) = \frac{1}{\sin(\alpha)} = \frac{1}{y}$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{y}{x} \qquad\qquad \cot(\alpha) = \frac{1}{\tan(\alpha)} = \frac{\cos(\alpha)}{\sin(\alpha)} = \frac{x}{y}$$

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