Math 75A **Maria Nogin**

Practice test 1 - Solutions

Multiple choice questions: circle the correct answer

1. The function $f(x) = \sin(x) + x^2$ is

A. even **B.** odd **C.** both even and odd **(D.)** neither even nor odd

2. If we shift the graph of $y = \sin(x)$ 2 units to the left, then the equation of the new graph is

A. $y = \sin(x) + 2$ **B.** $y = \sin(x) - 2$ **(C.)** $y = \sin(x + 2)$ **D.** $y = \sin(x - 2)$

E. $y = \sin(x/2)$

3. The domain of the function $f(x) = \dfrac{1}{\sqrt{x-1}}$ is the set of all real numbers x for which

A. $x > 0$ **B.** $x \neq 0$ **C.** $x \geq 1$ **(D.)** $x > 1$ **E.** $x \neq 1$

4. Simplify $\dfrac{1+x}{x} - \dfrac{\frac{1}{x}+1}{x+1}$.

(A.) 1 **B.** x **C.** $x + 1$ **D.** $\dfrac{1}{x}$ **E.** $\dfrac{x-1}{x+1}$

5. Let $f(x) = \begin{cases} -x-2 & \text{if } x < -1 \\ x-3 & \text{if } -1 \leq x \leq 1 \\ 2-x^2 & \text{if } x > 1 \end{cases}$. Find $f(1)$.

A. -3 **(B.)** -2 **C.** -1 **D.** 0 **E.** 1

6. If $f(x) = 1 + x$ and $g(x) = x^2 - 6$, find $(f \circ g)(-2)$.

A. -9 **B.** -7 **C.** -5 **(D.)** -1 **E.** Undefined

7. The function $f(x) = \begin{cases} -x-1 & \text{if } x < -1 \\ 0 & \text{if } -1 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases}$ is

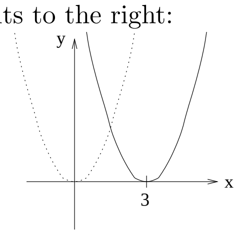
A. continuous everywhere
B. continuous at 1 but discontinuous at -1
(C.) continuous at -1 but discontinuous at 1
D. continuous at all points except for 1 and -1
E. discontinuous everywhere

(because there is a break in the graph at 1 but not at -1)

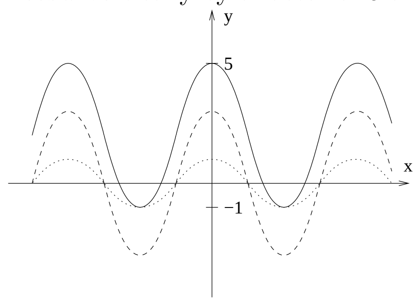
Regular problems: show all your work

8. Use transformations of functions to sketch the graphs of:

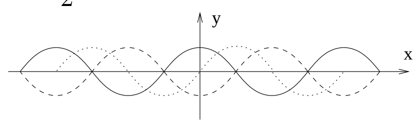
   (a) $(x-3)^2$
   Shift the curve $y = x^2$ 3 units to the right:



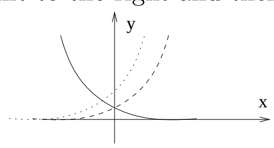
   (b) $3\cos x + 2$
   Stretch the curve $y = \cos x$ vertically by a factor of 3 and then shift 2 units upward:



   (c) $-\sin\left(x - \dfrac{\pi}{2}\right)$
   Shift the curve $y = \sin x$ $\dfrac{\pi}{2}$ units to the right and then reflect about the x-axis:



   (d) e^{-x-1}
   Shift the curve $y = e^x$ 1 unit to the right and then reflect about the y-axis:



9. Find a formula for the function whose graph is obtained from the graph of $f(x) = e^x - 1$ by

   (a) Reflecting about the y-axis and then compressing horizontally by a factor of 2.
   Reflecting about the y-axis: $y = e^{-x} - 1$
   Compressing horizontally by a factor of 2: $y = e^{-2x} - 1$

   (b) Vertically compressing by a factor of 5 and then shifting 3 units to the left.
   Vertically compressing by a factor of 5: $y = \dfrac{e^x - 1}{5}$
   Shifting 3 units to the left: $y = \dfrac{e^{x+3} - 1}{5}$

   (c) Reflecting about the x-axis and then shifting 2 units down.
   Reflecting about the x-axis: $y = -(e^x - 1) = -e^x + 1$
   Shifting 2 units down: $y = -e^x + 1 - 2 = -e^x - 1$

10. Let $f(x) = 2-x$, $g(x) = \frac{1}{x}$, $h(x) = \sqrt{x+1}$. Find the following functions and their domains:

(a) $(f+g)(x) = 2 - x + \frac{1}{x}$
Domain $= (-\infty, 0) \cup (0, \infty)$

(b) $(f-g)(x) = 2 - x - \frac{1}{x}$
Domain $= (-\infty, 0) \cup (0, \infty)$

(c) $(fg)(x) = (2-x) \cdot \frac{1}{x} = \frac{2-x}{x}$
Domain $= (-\infty, 0) \cup (0, \infty)$

(d) $\left(\frac{f}{g}\right)(x) = \frac{2-x}{\frac{1}{x}} = 2x - x^2$ (if $x \neq 0$)
Domain $= (-\infty, 0) \cup (0, \infty)$

(e) $(g \circ f)(x) = \frac{1}{2-x}$
Domain $= (-\infty, 2) \cup (2, \infty)$

(f) $(f \circ h)(x) = 2 - \sqrt{x+1}$
Domain $= [-1, \infty)$

(g) $(g \circ h)(x) = \frac{1}{\sqrt{x+1}}$
Domain $= (-1, \infty)$

(h) $(f \circ g \circ h)(x) = 2 - \frac{1}{\sqrt{x+1}}$
Domain $= (-1, \infty)$

11. Find the distance between $(-4, 3)$ and $(2, 11)$.
$D = \sqrt{(2-(-4))^2 + (11-3)^2} = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$

12. Write an equation of the circle

(a) whose radius is 3 and center is at $(3, -4)$
$(x-3)^2 + (y-(-4))^2 = 3^2$
$(x-3)^2 + (y+4)^2 = 9$

(b) whose center is at $(-2, 0)$ and that passes through the point $(1, 4)$
$r = \sqrt{(1-(-2))^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$
$(x-(-2))^2 + (y-0)^2 = 5^2$
$(x+2)^2 + y^2 = 25$

13. Write an equation of the line that

(a) has slope 2 and passes through the point $(-1, 3)$
$y - 3 = 2(x - (-1))$
$y - 3 = 2(x+1)$
$y - 3 = 2x + 2$
$y = 2x + 5$

(b) passes throught the points $(-1, 3)$ and $(0, -6)$

$m = \dfrac{-6-3}{0-(-1)} = \dfrac{-9}{1} = -9$

$y - 3 = -9(x - (-1))$

$y - 3 = -9(x + 1)$

$y - 3 = -9x - 9$

$y = -9x - 6$

(c) is parallel to the line $y = 7x - 1$ and passes through $(0, -6)$

$m = 7$

$b = -6$

$y = 7x - 6$

(d) is perpendicular to the line $y = 7x - 1$ and passes through $(0, -6)$

$m = -\dfrac{1}{7}$

$b = -6$

$y = -\dfrac{1}{7}x - 6$

14. Evaluate the following expressions:

(a) $\sin\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2}$

(b) $\cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

(c) $\tan\left(-\dfrac{\pi}{3}\right) = \dfrac{\sin\left(-\frac{\pi}{3}\right)}{\cos\left(-\frac{\pi}{3}\right)} = \dfrac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

(d) $\sec\left(\dfrac{3\pi}{4}\right) = \dfrac{1}{\cos\left(\frac{3\pi}{4}\right)} = \dfrac{1}{-\frac{\sqrt{2}}{2}} = -\dfrac{2}{\sqrt{2}} = -\sqrt{2}$

15. Evaluate the limits:

(a) $\lim_{x \to 5} (7x - 25) = 7 \cdot 5 - 25 = 10$

(b) $\lim_{x \to -1} \dfrac{x^3 + x^2}{x^2 + 3x + 2} = \lim_{x \to -1} \dfrac{x^2(x+1)}{(x+1)(x+2)} = \lim_{x \to -1} \dfrac{x^2}{x+2} = 1$

(c) $\lim_{x \to 0} \dfrac{3 - \sqrt{9+x}}{x} = \lim_{x \to 0} \dfrac{(3 - \sqrt{9+x})(3 + \sqrt{9+x})}{x(3 + \sqrt{9+x})} = \lim_{x \to 0} \dfrac{3^2 - (\sqrt{9+x})^2}{x(3 + \sqrt{9+x})} =$

$\lim_{x \to 0} \dfrac{9 - (9+x)}{x(3 + \sqrt{9+x})} = \lim_{x \to 0} \dfrac{-x}{x(3 + \sqrt{9+x})} = \lim_{x \to 0} \dfrac{-1}{3 + \sqrt{9+x}} = -\dfrac{1}{6}$

(d) $\lim_{x \to 0} x^4 \cos\left(\dfrac{1}{x}\right) = 0$ by the squeeze theorem since $-x^4 \leq x^4 \cos\left(\dfrac{1}{x}\right) \leq x^4$ and $\lim_{x \to 0} (-x^4) = \lim_{x \to 0} (x^4) = 0$.

16. Show that the equation $x^5 - 4x + 2 = 0$ has at least one solution in the interval $(1, 2)$.

Let $f(x) = x^5 - 4x + 2$. Then $f(x)$ is a continuous function with $f(1) = -1 < 0$ and $f(2) = 26 > 0$. By the intermediate value theorem, there is a point c between 1 and 2 such that $f(c) = 0$.

17. Find all values of c such that the function $f(x)$ is continuous everywhere.

(a) $f(x) = \begin{cases} cx & \text{if} \quad x \geq 2 \\ 5 - x & \text{if} \quad x < 2 \end{cases}$

Since linear functions are continuous everywhere, $f(x)$ is continuous at all poits except possibly at 2. It is continuous at 2 if and only if the functions cx and $5 - x$ agree at 2 (that is, they have the same value at 2. The graph of $f(x)$ then has no jump at 2.) So we set the values of cx and $5 - x$ at 2 equal:

$c \cdot 2 = 5 - 2$

$2c = 3$

$c = \frac{3}{2}$

(b) $f(x) = \begin{cases} x^2 & \text{if} \quad x \leq c \\ x^3 & \text{if} \quad x > c \end{cases}$

Since polynomial functions are continuous everywhere, $f(x)$ is continuous at all poits except possibly at c. It is continuous at c if and only if the values of x^2 and x^3 agree at c, i.e.

$c^2 = c^3$

$c^2 - c^3 = 0$

$c^2(1 - c) = 0$

$c = 0$ or $c = 1$.