MATH 75A

Test 2 - Solutions

Multiple choice questions: circle the correct answer

1. Solve for x: $2^{x-1} = \frac{1}{8}$

   (A.) -2 **B.** $-\frac{4}{3}$ **C.** $\frac{4}{3}$ **D.** $1\frac{1}{16}$ **E.** 4

2. How many horizontal asymptotes does the curve $y = \frac{x+2}{(x+1)(x+3)}$ have?

   A. 0 **(B.)** 1 **C.** 2 **D.** 3 **E.** 4

3. Evaluate $\lim_{x\to\infty} \frac{2x^2+7x+3}{5x^2-3x+4}$.

   A. 0 **(B.)** $\frac{2}{5}$ **C.** $\frac{3}{4}$ **D.** 1 **E.** Does not exist

4. Evaluate $\lim_{x\to-\infty} \frac{2x^2+8x-2}{7x^3-2x-4}$.

   (A.) 0 **B.** $\frac{2}{7}$ **C.** $\frac{1}{2}$ **D.** 1 **E.** Does not exist

5. If $f(x) = 7$, find $f'(2)$.

   (A.) 0 **B.** 2 **C.** 4 **D.** 7 **E.** 14

6. If $f(x) = 3x + 2$, find $f'(4)$.

   A. 0 **B.** 2 **(C.)** 3 **D.** 8 **E.** 14

Regular problems: show all your work

7. Evaluate the limit: $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$.

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$$

8. Find the vertical asymptotes of $f(x) = \dfrac{x^3-4x}{x^2-3x+2}$.

The given function is undefined when $x^2-3x+2=0$, or $(x-2)(x-1)=0$, i.e. at $x=2$ and $x=1$.

Since $f(x) = \dfrac{x^3-4x}{x^2-3x+2} = \dfrac{x(x^2-4)}{(x-2)(x-1)} = \dfrac{x(x-2)(x+2)}{(x-2)(x-1)} = \dfrac{x(x+2)}{x-1}$,

$\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{x(x+2)}{x-1} = 8$ *which is a finite number, therefore $x=2$ is not a vertical asymptote.*

$\lim_{x\to 1^+} f(x) = \lim_{x\to 2} \dfrac{x(x+2)}{x-1} = \infty$, *therefore $x=1$ is a vertical asymptote.*

Answer: $x=1$ is the only vertical asymptote.

9. Show that the equation $x^3+4x+2=0$ has a solution in the interval $(-1,1)$.

Let $f(x) = x^3+4x+2$. Then $f(-1) = -3 < 0$ and $f(1) = 7 > 0$. Since f is a polynomial, it is continuous everywhere. In particular, it is continuous on $[-1,1]$. Therefore by the IVT (Intermediate Value Theorem) the function f has a root in $(-1,1)$.

10. Find all values of c such that the function $f(x) = \begin{cases} cx & \text{if } x<4 \\ x+6 & \text{if } x\geq 4 \end{cases}$ is continuous everywhere.

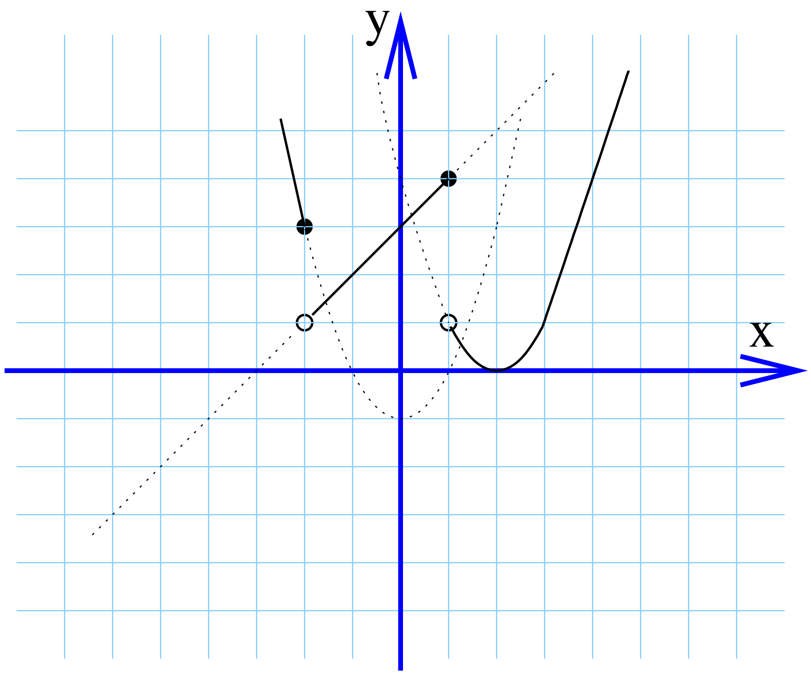
The function f is continuous everywhere except possibly at 4 since linear functions are continuous everywhere. It is continuous at 4 if and only if the functions cx and $x+6$ agree at $x=4$, i.e.

$c\cdot 4 = 4+6$

$4c = 10$

$c = \dfrac{10}{4} = 2.5$

11. (a) Sketch the graph of $f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq -2 \\ x + 3 & \text{if } -2 < x \leq 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$.



(Note: the dotted lines and curves are shown so that it is easier to see the line and the parabolas, but they are not a part of this graph.)

(b) At which point(s) is this function discontinuous?

It is discontinuous at 1 and at -2.

(c) At the above point(s), is $f(x)$ continuous from the right, continuous from the left, or neither?

Continuous from the left at both 1 and -2.