MATH 75A

Test 3 - Solutions

Multiple choice questions: circle the correct answer

1. Let $f(x) = 5x^3 - 4x^2$. Find $f'(-1)$.

   A. -9 **B.** -7 **C.** 1 **D.** 7 (**E.**) 23

2. Let $f(x) = 2\tan x$. Find $f'(0)$.

   A. 0 **B.** $\frac{1}{2}$ (**C.**) 2 **D.** 4 **E.** Does not exist

3. Simplify the expression: $\dfrac{(2x)^3 - 5x^3}{6x^2\sqrt{x}}$.

   A. $-\frac{\sqrt{x}}{2}$ (**B.**) $\frac{\sqrt{x}}{2}$ **C.** $\frac{1}{2\sqrt{x}}$ **D.** $\frac{2}{\sqrt{x}}$ **E.** $\frac{3x^{-1/2}}{6}$

4. The position of an object at time t is given by $s(t) = 6\cos t + 2\sin t$. Find the velocity of this object at $t = \frac{\pi}{6}$.

   (**A.**) $\sqrt{3} - 3$ **B.** $-3 - \sqrt{3}$ **C.** $1 - 3\sqrt{3}$ **D.** $3 + \sqrt{3}$ **E.** $3\sqrt{3} + 1$

5. If $f(3) = 2$, $f'(3) = 4$, $g(3) = 5$, and $g'(3) = 6$, find the derivative of $f(x)g(x)$ at $x = 3$.

   A. 2 **B.** 10 **C.** 24 (**D.**) 32 **E.** 34

6. If $f(x) = 3^{2-x}$, find $f'(x)$.

   A. 3^{2-x} **B.** -3^{2-x} **C.** $\ln(3)3^{2-x}$ (**D.**) $-\ln(3)3^{2-x}$ **E.** None of these

Regular problems: show all your work

7. Differentiate the following functions:

   (a) $f(x) = 6x^4 - \dfrac{5}{\sqrt[3]{x}} + 2e^x$

   $$f(x) = 6x^4 - 5x^{-\frac{1}{3}} + 2e^x$$
   $$f'(x) = 24x^3 + \frac{5}{3}x^{-\frac{4}{3}} + 2e^x$$

   (b) $g(x) = \pi^3 - 2\sin(x^3)$

   $$g'(x) = -2\cos(x^3)3x^2 = -6x^2\cos(x^3)$$

8. Find the points where the tangent line to the curve $y = \dfrac{x^2 - 3}{x - 2}$ is horizontal.

   The tangent line is horizontal when the derivative is equal to 0 *(because the slope of a horizontal line is* 0*).*

   $$y' = \frac{2x(x-2) - (x^2 - 3)}{(x-2)^2} = \frac{x^2 - 4x + 3}{(x-2)^2}$$
   $$\frac{x^2 - 4x + 3}{(x-2)^2} = 0$$
   $$x^2 - 4x + 3 = 0$$
   $$(x-1)(x-3) = 0$$
   $x = 1$ *or* $x = 3$.

   If $x = 1$ *then* $y = \dfrac{-2}{-1} = 2$*; if* $x = 3$ *then* $y = \dfrac{6}{1} = 6$.

   Thus there are two points where the tangent line is horizontal: $(1, 2)$ *and* $(3, 6)$.

9. Find an equation of the tangent line to $y = \sqrt{x^2 - 9}$ at $(5, 4)$.

   $$y' = \frac{1}{2}(x^2 - 9)^{-\frac{1}{2}}2x = \frac{x}{\sqrt{x^2 - 9}}$$
   $$y'(5) = \frac{5}{4}$$

   Therefore the slope of the tangent line is $\dfrac{5}{4}$*, and an equation is:*

   $$y - 4 = \frac{5}{4}(x - 5)$$
   $$y = \frac{5}{4}x - \frac{25}{4} + 4$$
   $$y = \frac{5}{4}x - \frac{9}{4}$$

10. Evaluate the limits:

(a) $\lim_{x\to 0} \dfrac{\sin(2x)}{3x} = \lim_{x\to 0} \dfrac{\sin(2x)}{3\cdot\frac{1}{2}\cdot 2x} = \dfrac{1}{3/2} = \dfrac{2}{3}$

(b) $\lim_{x\to 0} \cot(2x)\sin(3x) = \lim_{x\to 0} \dfrac{\cos(2x)\sin(3x)}{\sin(2x)} = \lim_{x\to 0} \dfrac{\cos(2x)\frac{\sin(3x)}{x}}{\frac{\sin(2x)}{x}} = \dfrac{\lim_{x\to 0}\cos(2x)\dfrac{\sin(3x)}{x}}{\lim_{x\to 0}\dfrac{\sin(2x)}{x}} =$

$$\frac{\lim_{x\to 0}\cos(2x)\dfrac{\sin(3x)}{\frac{1}{3}\cdot 3x}}{\lim_{x\to 0}\dfrac{\sin(2x)}{\frac{1}{2}\cdot 2x}} = \frac{\dfrac{1}{1/3}}{\dfrac{1}{1/2}} = \frac{3}{2}$$

11. The size of a bacteria population at time t is $P = 100(e^t - t)$, where time is measured in days. Find the rate of growth of the population at $t = 4$.

The rate of growth is the derivative: $P'(t) = 100(e^t - 1)$. *When* $t = 4$ *we have:*
$P'(4) = 100(e^4 - 1)$.

12. Find an equation of the tangent line to the curve $x^4 - y^4 - 2x^3y = -11$ at $(-1, 2)$.

$x^4 - (y(x))^4 - 2x^3y(x) = -11$

$4x^3 - 4(y(x))^3y'(x) - (6x^2y(x) + 2x^3y'(x)) = 0$

$4x^3 - 4y^3y' - 6x^2y - 2x^3y' = 0$

At the point $(-1, 2)$, $x = -1$ *and* $y = 2$, *so*

$-4 - 32y' - 12 + 2y' = 0$

$-30y' - 16 = 0$

$-30y' = 16$

$y' = \dfrac{16}{-30} = -\dfrac{8}{15}$

Therefore the slope of the tangent line is $-\dfrac{8}{15}$.

An equation is:

$y - 2 = -\dfrac{8}{15}(x + 1)$

$y = -\dfrac{8}{15}x - \dfrac{8}{15} + 2$

$y = -\dfrac{8}{15}x + \dfrac{22}{15}$