Math 75B **Maria Nogin**

Practice test 2

The actual exam will consist of 6 multiple choice questions and 6 regular problems. You will have 1 hour to complete the exam.

Multiple choice questions: circle the correct answer

1. If a bacteria population has initial size of 100 and doubles every hour, what is, approximately, the size of the population after 2.5 hours?

   A. 255 **B.** 300 **C.** 450 **D.** 566 **E.** 600

2. Find the exact value of $\arccos\left(-\frac{1}{2}\right)$.

   A. $-\frac{\pi}{3}$ **B.** $-\frac{\pi}{6}$ **C.** $\frac{\pi}{6}$ **D.** $\frac{\pi}{3}$ **E.** $\frac{2\pi}{3}$

3. Find the derivative of $\arctan x + \arctan 2$.

   A. $\frac{1}{1+x^2}$ **B.** $\frac{1}{6+x^2}$ **C.** $\frac{1}{1+x^2}+\frac{1}{5}$ **D.** $\frac{1}{(1+x^2)5}$
   E. none of the above

4. If $f(1) = 3$ and $f'(x) \le 2$, which of the following must be true about $f(4)$?

   A. $f(4) \le 6$ **B.** $f(4) \le 9$ **C.** $f(4) \le 11$ **D.** $f(4) \ge 5$ **E.** $f(4) \ge 8$

5. Use Newton's method to approximate the root of the equation $x^2 - 23 = 0$. Let $x_1 = 5$. Find x_2.

   A. 0 **B.** 0.3 **C.** 4.8 **D.** 5 **E.** 5.2

6. Which of the following functions is an antiderivative of $\sqrt{x}$?

   A. $2\sqrt{x}$ **B.** $\frac{1}{2\sqrt{x}}$ **C.** $\frac{3}{2}x^{\frac{3}{2}}$ **D.** $\frac{2x\sqrt{x}}{3}$
   E. none of the above

Regular problems: show all your work

7. Consider the curve given by $x^2y + 3xy + 4xy^2 = 102$. Find an equation of the tangent line to this curve at the point $(2, 3)$.

8. If both legs of a right triangle are increasing at a rate of 4 cm/min, how fast is the hypothenuse increasing when the legs are 5 cm and 12 cm long?

9. Find the absolute maximum and minimum values of $f(x) = x + \frac{4}{x^2}$ on the interval $[1, 4]$.

10. Evaluate the limits:

    (a) $\lim_{x\to\infty} \dfrac{x}{e^x}$

    (b) $\lim_{x\to 0} \dfrac{\sin(2x)}{\tan(3x)}$

11. Let $f(x) = x^2 e^x$. Find the following:

    (a) domain

    (b) intercepts

    (c) vertical and horizontal asymptotes

    (d) critical numbers

    (e) intervals of increase and decrease

    (f) local and absolute maxima and minima

    (g) intervals of concavity

    (h) inflection points

    (i) sketch the graph of $f(x)$

12. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side 10 if one side of the rectangle lies on the base of the triangle.

13. Find $f(x)$ if

    (a) $f'(x) = 1 - 8x^3 + 2\sin x - \cos x,\ f(0) = 5.$

    (b) $f''(x) = 6 - 24x^2,\ f'(1) = -3,\ f(2) = -32.$