MATH 75B

Test 3 - Solutions

1. C

   $$\int_{-1}^{1} x^2 dx = \left.\frac{x^3}{3}\right|_{-1}^{1} = \frac{1^3}{3} - \frac{(-1)^3}{3} = \frac{2}{3}$$

2. A

   $$\left(\int_x^3 \cos(\sqrt{t})dt\right)' = \left(-\int_3^x \cos(\sqrt{t})dt\right)' = -\cos(\sqrt{x}).$$

3. C

   Draw graphs of a few increasing functions and try all choices of sample points (draw rectangles).

4. C

   $\Delta x = \frac{b-a}{n} = \frac{9-(-3)}{6} = 2$

5. C

   To see that C is correct, draw a graph of any function (e.g. $f(x) = e^x$ would work well for this example), draw the graph of $f(-x)$, and interpret both integrals in terms of areas. A and B are incrrect as was discussed a number of times in class. D is incorrect; in fact, by definition $\displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$. E is incorrect as can be seen e.g. for $f(x) = e^x$.

6. D

   Draw all the lines, identify the region, and divide the region into trapezoids and/or rectangles and/or triangles - there are multiple ways of doing that.

7. For n subintervals we have $\Delta x = \frac{3}{n}$ and $x_i = \frac{3}{n}i$.

   Then $R_n = \displaystyle\sum_{i=1}^{n}\left(2\left(\frac{3}{n}i\right)\frac{3}{n}\right) = \sum_{i=1}^{n}\left(\frac{18}{n^2}i\right) = \frac{18}{n^2}\sum_{i=1}^{n} i = \frac{18}{n^2}\cdot\frac{n(n+1)}{2} = \frac{18n(n+1)}{2n^2} = \frac{9(n+1)}{n}$.

Now, $\displaystyle\int_0^3 2x\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{9(n+1)}{n} = 9.$

8. (a) This integral represents the area of the triangle (sketch it) with base 3 and height 6. The area is 9.

   (b) $\displaystyle\int_0^3 2x\,dx = x^2|_0^3 = 3^2 - 0^2 = 9.$

9. Using the substitution $u = x^3$, $\frac{du}{dx} = 3x^2$, $\frac{1}{3}du = x^2dx$:

   $$\int x^2 \sin(x^3)dx = \frac{1}{3}\int \sin(u)du = -\frac{1}{3}\cos(u) + C = -\frac{1}{3}\cos(x^3) + C$$

10. Since $y = \sqrt{4 - x^2}$ is the equation of the top half of the circle with center at the origin and radius 2,

    $\displaystyle\int_{-2}^0 \sqrt{4-x^2}dx$ represents the area of 1/4 of the circle, so $\displaystyle\int_{-2}^0 \sqrt{4-x^2}dx = \frac{1}{4}\pi(2)^2 = \pi.$

11. The acceleration is $a(t) = \frac{v(10)-v(0)}{10} = \frac{0-60}{10} = -6$. So we have $a(t) = -6$ and $v(0) = 60$. By antidifferentiating we find $v(t) = -6t + C$. Since $v(0) = 60$, $C = 60$, so $v(t) = -6t + 60$. Then distance can be found by

    $\displaystyle\int_0^{10} v(t)dt = \int_0^{10}(-6t+60)dt = (-3t^2+60t)|_0^{10} = (-3(10)^2+60(10))-0 = -300+600 =$ 300 ft.

12. First, $\displaystyle\int_0^{10} 2f(x)dx = 18$ implies that $\displaystyle\int_0^{10} f(x)dx = 9$. Next, together with $\displaystyle\int_0^5 f(x) = 6dx$, this gives $\displaystyle\int_5^{10} f(x)dx = 9 - 6 = 3$. Finally, $\displaystyle\int_5^{10}(f(x)+1)dx = \int_5^{10} f(x)dx + \int_5^{10} 1dx = 3 + 5 = 8.$