

Practice test 1

The actual exam will consist of 6 multiple choice questions and 6 regular problems.
You will have 1 hour to complete the exam.

Multiple choice questions: circle the correct answer

- Find the exact value of $\arcsin(1)$.
A. 0 B. $\frac{\pi}{2}$ C. π D. $\frac{3\pi}{2}$ E. 2π
- Find the exact value of $\arccos\left(\frac{1}{2}\right)$.
A. 0 B. $\frac{\pi}{6}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{3}$ E. $\frac{\pi}{2}$
- Find the exact value of $\sin\left(\arctan\left(\frac{3}{4}\right)\right)$.
A. $-\frac{3}{5}$ B. $-\frac{3}{4}$ C. $\frac{3}{5}$ D. $\frac{3}{4}$ E. $\frac{4}{5}$
- Suppose 100 dollars are invested at an annual interest rate of 10% while interest is compounded monthly. What is the amount after 10 years?
A. $100\left(1 + \frac{1}{120}\right)^{10}$ B. $100\left(1 + \frac{1}{120}\right)^{120}$ C. $100\left(1 + \frac{10}{12}\right)^{10}$
D. $120\left(1 + \frac{10}{12}\right)^{100}$ E. $120\left(1 + \frac{1}{120}\right)^{100}$
- How many critical numbers does the function $y = x + \frac{1}{x}$ have?
A. 0 B. 1 C. 2 D. 3 E. infinitely many
- Find the local maximum of $y = x + \frac{1}{x}$.
A. $x = -2$ B. $x = -1$ C. $x = 0$ D. $x = 1$ E. $x = 2$

Regular problems: show all your work

- Consider the curve given by $x^3y^3 - 3xy^3 + 4y = 6$.
 - Use implicit differentiation to find $y'(x)$.
 - Check that the point $(2, 1)$ lies on this curve.
 - What is the slope of the tangent line to this curve at $(2, 1)$?
- Find the slope of the tangent line to the curve $x \tan y + xy + 3y = 0$ at the point $(0, 0)$.

9. A boy starts walking west at 6 km/h from a point P . Five minutes later a girl starts walking
- north
 - east
- at 4 km/h from a point 15 km due south from P . At what rate is the distance between the kids changing 45 min after the girl starts walking? Is the distance increasing or decreasing at this instant?
10. A snowball is melting so that its radius is decreasing at a rate of 1 cm/min. Find the rate at which its volume is decreasing when the radius is 3 cm.
11. A bacteria cultute starts with 800 bacteria and the growth rate is proportional to the number of bacteria. After 3 hours the population is 2700. Find the number of bacteria after 5 hours.
12. Differentiate the following functions.
- $f(x) = \arcsin(3x)$
 - $g(x) = x \tan^{-1}(1 - x)$
 - $h(x) = \frac{\arccos(x)}{\sqrt{1-x^2}}$
13. Evaluate the following limits.
- $\lim_{x \rightarrow 0} \frac{\sin 5x}{2 \sin 3x}$
 - $\lim_{x \rightarrow 0} \frac{e^x(\cos x - 1)}{\tan(3x)}$
 - $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$
 - $\lim_{x \rightarrow \infty} x^3 e^{-3x}$
 - $\lim_{x \rightarrow \infty} \left(\frac{x}{x+1} \right)^{3x}$