Math 75B **Maria Nogin**

Practice test 1 - Solutions

Multiple choice questions: circle the correct answer

1. Find the exact value of $\arcsin(1)$.

   A. 0 **B.** $\frac{\pi}{2}$ **C.** π **D.** $\frac{3\pi}{2}$ **E.** 2π

2. Find the exact value of $\arccos\left(\frac{1}{2}\right)$.

   A. 0 **B.** $\frac{\pi}{6}$ **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{3}$ **E.** $\frac{\pi}{2}$

3. Find the exact value of $\sin\left(\arctan\left(\frac{3}{4}\right)\right)$.

   A. $-\frac{3}{5}$ **B.** $-\frac{3}{4}$ **C.** $\frac{3}{5}$ **D.** $\frac{3}{4}$ **E.** $\frac{4}{5}$

4. Suppose 100 dollars are invested at an annual intererst rate of 10% while interest is compounded monthly. What is the ammount after 10 years?

   A. $100\left(1+\frac{1}{120}\right)^{10}$ **B.** $100\left(1+\frac{1}{120}\right)^{120}$ **C.** $100\left(1+\frac{10}{12}\right)^{10}$
   D. $120\left(1+\frac{10}{12}\right)^{100}$ **E.** $120\left(1+\frac{1}{120}\right)^{100}$

5. How many critical numbers does the function $y = x + \dfrac{1}{x}$ have?

   A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** infinitely many

6. Find the local maximum of $y = x + \dfrac{1}{x}$.

   A. $x=-2$ **B.** $x=-1$ **C.** $x=0$ **D.** $x=1$ **E.** $x=2$

Regular problems: show all your work

7. (a) $3x^2y^3 + 3x^3y^2y' - 3y^3 - 9xy^2y' + 4y' = 0$

   $(3x^3y^2 - 9xy^2 + 4)y' = 3y^3 - 3x^2y^3$

   $$y' = \frac{3y^3 - 3x^2y^3}{3x^3y^2 - 9xy^2 + 4}$$

   (b) $2^3 - 3\cdot 2 + 4 = 6$

   (c) $$y'(2) = \frac{3 - 3\cdot 2^2}{3\cdot 2^3 - 9\cdot 2 + 4} = -\frac{9}{10}$$

8. $\tan y + x\sec^2 y \cdot y' + y + xy' + 3y' = 0$

   $(x\sec^2 y + x + 3)y' = -\tan y - y$

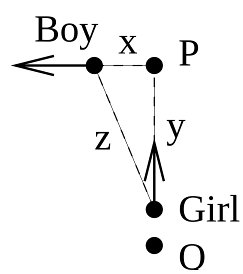
   If $x = 0$ and $y = 0$, then $3y'(0) = 0$, so the slope of the tangent line is 0.

9. (a) Let x be the distance between the boy and the point P, let y be the distance between the girl and P, and let z be the distance between the boy and the girl. Then $x^2 + y^2 = z^2$ where x, y, and z are functions of time.

   Differentiating this equation with respect to t gives

   $2xx' + 2yy' = 2zz'$

   $xx' + yy' = zz'$

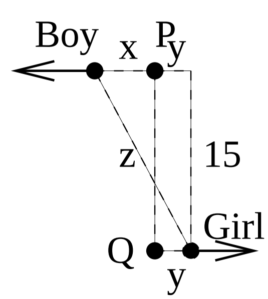


   45 minutes after the girl started walking (and thus 50 minutes after the boy started walking), $x = 6 \cdot \frac{50}{60} = 5$, $y = 15 - 4 \cdot 4560 = 15 - 3 = 12$, and $z = \sqrt{5^2 + 12^2} = 13$. x' is the rate of change of x, i.e. the speed of the boy, so $x' = 6$, and y' is the rate of change of y, i.e. negative the speed of the girl since y is decreasing, so $y' = -4$. Therefore

   $5 \cdot 6 + 12 \cdot (-4) = 13z$

   Answer: $-\dfrac{18}{13}$, decreasing.

   (b) Let x be the distance between the boy and the point P, let y be the distance between the girl and her starting point Q, and let z be the distance between the boy and the girl.



   Then $(x + y)^2 + 15^2 = z^2$ (see the figure)

   Differentiating this equation with respect to t gives

   $2(x + y)(x' + y') = 2zz'$

   $(x + y)(x' + y') = zz'$

   45 minutes after the girl started walking (and thus 50 minutes after the boy started walking), $x = 6 \cdot \frac{50}{60} = 5$, $y = 4 \cdot 4560 = 3$, so $x + y = 8$, and $z = \sqrt{8^2 + 15^2} = 17$. x' is the rate of change of x, i.e. the speed of the boy, so $x' = 6$, and y' is the rate of change of y, i.e. the speed of the girl, so $y' = 4$. Therefore

   $(5 + 3)(6 + 4) = 17z'$

   Answer: $\dfrac{80}{17}$, increasing.

10. $V(t) = \frac{4}{3}\pi(r(t))^3$

$V'(t) = 4\pi(r(t))^2 r'(t)$

If $r' = -1$ and $r = 3$, $V'(t) = 4\pi 3^2 \cdot 1 = 36\pi$

Answer: 36π cm^3/min.

11. Since initially there are 800 bacteria, $P(t) = 800e^{kt}$. At $t = 3$ we have: $2700 = 800e^{k\cdot 3}$ $(e^k)^3 = \frac{27}{8}$ $e^k = \frac{3}{2}$. Then at $t = 5$: $P(5) = 800e^{k\cdot 5} = 800(e^k)^5 = 800\left(\frac{3}{2}\right)^5 = \frac{800\cdot 3^5}{2^5} = 25 \cdot 343 = 6075$.

12. (a) $f'(x) = \frac{3}{\sqrt{1-9x^2}})$

(b) $'g(x) = \tan^{-1}(1-x) + \frac{-x}{1+(1-x)^2}$

(c) $h(x) = \frac{-\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} - \arccos(x)\frac{-2x}{2\sqrt{1-x^2}}}{1-x^2} = \frac{-1+\frac{x\arccos(x)}{\sqrt{1-x^2}}}{1-x^2}$

13. (a) $\lim_{x\to 0} \frac{\sin 5x}{2\sin 3x} = \lim_{x\to 0} \frac{5\cos 5x}{6\cos 3x} = \frac{5}{6}$

(b) $\lim_{x\to 0} \frac{e^x(\cos x - 1)}{\tan(3x)} = 1 \cdot \lim_{x\to 0} \frac{\cos x - 1}{\tan(3x)} = \lim_{x\to 0} \frac{-\sin x}{3\sec^2(3x)} = 0$

(c) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x\to 0} \frac{e^x - 1}{2x} = \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$

(d) $\lim_{x\to\infty} x^3 e^{-3x} = \lim_{x\to\infty} \frac{x^3}{e^{3x}} = \lim_{x\to\infty} \frac{3x^2}{3e^{3x}} = \lim_{x\to\infty} \frac{x^2}{e^{3x}} = \lim_{x\to\infty} \frac{2x}{3e^{3x}} = \lim_{x\to\infty} \frac{2}{9e^{3x}} = 0$

(e) $\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^{3x} = \lim_{x\to\infty} \left(e^{\ln\frac{x}{x+1}}\right)^{3x} = e^{\lim_{x\to\infty} \ln\frac{x}{x+1} \cdot 3x} = e^{\lim_{x\to\infty} \frac{3\ln\frac{x}{x+1}}{1/x}} =$
$e^{\lim_{x\to\infty} \frac{3\frac{x+1}{x} \cdot \frac{1}{(x+1)^2}}{-\frac{1}{x^2}}} = e^{\lim_{x\to\infty} \frac{\frac{3}{x(x+1)}}{-\frac{1}{x^2}}} = e^{\lim_{x\to\infty} -\frac{3x^2}{x(x+1)}} = e^{\lim_{x\to\infty} -\frac{3x}{x+1}} = e^{-3}$