

MATH 75B

Test 1

September 30, 2009

**This test is to be taken on a furlough day.
It is take-home, self-check, and not part of your grade.**

Name: _____

- No books or calculators are allowed.
- You may use the unit circle. However, no other notes are allowed.
- Please show all your work for problems 7-12.

Multiple choice questions: circle the correct answer

1. If the size of a bacteria population is described by the formula $P(t) = Ce^{kt}$, what is the meaning of the constant C ?

A. rate of growth of the population

B. relative rate of growth

C. size of the population at $t = 0$

D. size of the population at $t = 1$

E. none of the above

2. Find the exact value of $\arccos\left(-\frac{\sqrt{3}}{2}\right)$.

A. $-\frac{\pi}{6}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

E. $\frac{5\pi}{6}$

3. Find the exact value of $\arcsin(\sin(\pi))$.

A. $-\pi$

B. $-\frac{\pi}{2}$

C. 0

D. $\frac{\pi}{2}$

E. π

4. Evaluate the limit: $\lim_{x \rightarrow \infty} \frac{3x}{e^x}$

A. -1

B. 0

C. 1

D. 3

E. ∞

5. Find the critical numbers of the function $f(x) = x^3 - 12x$.

A. 0

B. 2

C. 12

D. $\sqrt[3]{12}$

E. ± 2

6. The Mean Value Theorem can be used to show that the equation $x^3 + x + 1 = 0$

A. has no real roots

B. has at least one real root

C. has at most one real root

D. has at least two real roots

E. has infinitely many real roots

Regular problems: show all your work

7. Consider the curve given by $x^2y + xy^3 = 4x - 2$.

(a) Use implicit differentiation to find $y'(x)$.

(b) Check that the point $(-1, 2)$ lies on this curve.

(c) What is the slope of the tangent line to this curve at $(-1, 2)$?

8. At noon, ship A is 100 km west of ship B. Ship A is sailing east at 30 km/h and ship B is sailing north at 20 km/h. How fast is the distance between them changing at 3 PM?

9. A sample of tritium-3 decayed to 94.5% of its original amount after a year. What is the half-life of tritium-3?

10. Differentiate the following functions.

(a) $f(x) = \sqrt{x} \arcsin(x)$

(b) $g(x) = \arctan\left(\frac{1}{x}\right)$

11. Evaluate the following limit: $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$

12. Find the absolute maximum and minimum values of $f(x) = \frac{x}{x^2 + 9}$ on the interval $[-1, 9]$.