Math 75 **Fall 2003**

Practice Final

The exam will consist of 25 multiple choice questions.

You will have 2 hours to complete the exam.

1. If $f(x) = (\sqrt[3]{x})^2$, then $\dfrac{df}{dx} =$
   (a) $\frac{2}{3}x^{5/3}$
   (b) $\frac{2}{3x^{1/3}}$
   (c) $\frac{2}{3x^{-1/3}}$
   (d) $\frac{3}{5}x^{5/3}$
   (e) $\frac{3}{2}\sqrt{x}$
2. The vertical and horizontal asymptotes for the function $f(x) = \dfrac{3-x^2}{x^2-9}$ are
   (a) $x = 3$, $x = -3$, $y = -1$
   (b) $x = 3$, $y = -1$
   (c) $x = -1$, $y = 3$, $y = -3$
   (d) $x = -1$, $y = -3$
   (e) $x = 3$, $x = -3$
3. The derivative of $f(x) = \displaystyle\int_1^x \sin(1+t^4)dt$ is:
   (a) $\sin(1+x^4)$
   (b) $4x^3\sin(1+x^4)$
   (c) $4x^3\cos(1+x^4)$
   (d) $-\sin(1+x^4)$
   (e) $-\cos(1+x^4)$
4. The absolute minimum value of the function $f(x) = \sec x$ on the interval $[-1, 1]$ is
   (a) 0
   (b) -1
   (c) 1
   (d) -2
   (e) 2
5. Which of the following is the linear approximation of the function $f(x) = \dfrac{1}{\sqrt{x}}$ near the number $a = 1$?
   (a) $2x - 1$
   (b) $-2x + 3$
   (c) $\frac{1}{2}x + \frac{1}{2}$
   (d) $-\frac{1}{2}x + 1$
   (e) $-\frac{1}{2}x + \frac{3}{2}$

6. Which of the following statements is true?
   (a) If f and g are increasing functions on an interval I, then fg is increasing on I.
   (b) If $f'(c) = 0$, then $f(x)$ has a local minimum or a local maximum at c.
   (c) If $f''(x) = 0$, then c is an inflection point of $f(x)$.
   (d) All continuous functions are differentiable.
   (e) All differentiable functions are continuous.

7. Find the value of k for which the function $f(x) = \begin{cases} \dfrac{x-9}{\sqrt{x}-3} & x \neq 9 \\ k & x = 9 \end{cases}$ is continuous at $x = 9$:
   (a) 0
   (b) -3
   (c) 3
   (d) 9
   (e) 6

8. $\displaystyle\int_2^{10} |x-4|dx =$
   (a) 0
   (b) 16
   (c) 18
   (d) 20
   (e) 40

9. A poster is to have an area of 180 in^2 with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions of the poster will give the largest printed area?
   (a) width $= 5\sqrt{5}$, height $= 6\sqrt{5}$
   (b) width $= 6\sqrt{5}$, height $= 5\sqrt{5}$
   (c) width $= 2\sqrt{30}$, height $= 3\sqrt{30}$
   (d) width $= 10$, height $= 15$
   (e) width $= 15$, height $= 10$

10. $\displaystyle\int (2 + 3\sin x + 4\cos x)dx =$
    (a) $3\cos x - 4\sin x$
    (b) $3\cos x - 4\sin x + C$
    (c) $2x + 3\cos x - 4\sin x + C$
    (d) $2x - 3\cos x + 4\sin x + C$
    (e) $2x + 3\cos x + 4\sin x + C$

11. The inflection points of the function $y = 2x^6 - 3x^5 - 10x^4 + 11$ are
    (a) $(0, 11)$ only
    (b) $(-1, 6)$ and $(2, 75)$
    (c) $(-1, 6)$, $(0, 1)$, and $(2, 75)$
    (d) $(1, 0)$ only
    (e) $(0, 11)$ and $(-1, 6)$

12. If $y = \cos(\cot x)$, then $\dfrac{dy}{dx} =$
    (a) $-\sin(\cot x)$
    (b) $-\sin(-\csc x \cot x)$
    (c) $-\sin x \cot x - \cos x \csc x \cot x$
    (d) $\dfrac{\cot x \cos x}{\sin x}$
    (e) $\dfrac{\sin(\cot x)}{\sin^2 x}$
13. Which of the following is equal to the area under the curve $y = |x^2 - 4|$ between $x = 0$ and $x = 4$?
    (a) $\displaystyle\int_0^4 (x^2 - 4)dx$
    (b) $\displaystyle -\int_0^4 (x^2 - 4)dx$
    (c) $\displaystyle\int_2^4 (x^2 - 4)dx$
    (d) $\displaystyle\int_0^2 (x^2 - 4)dx + \int_2^4 (4 - x^2)dx$
    (e) $\displaystyle\int_0^2 (4 - x^2)dx + \int_2^4 (x^2 - 4)dx$
14. $\displaystyle\lim_{x \to 2} \frac{2x^2 - 5x + 2}{x^2 - x - 2} =$
    (a) 0
    (b) 1
    (c) 2
    (d) ∞
    (e) $-\infty$
15. $\displaystyle\int_0^3 x \sin(x^2 - 2)dx =$
    (a) $\frac{1}{2}\cos(2) - \frac{1}{2}\cos(7)$
    (b) $-\frac{1}{2}\cos(7) - \frac{1}{2}\cos(2)$
    (c) $1 - \cos(3)$
    (d) $\cos(-2) - \cos(7)$
    (e) $-\frac{1}{2}(\cos(3) - 1)$
16. If $F(x) = f(g(x))$, $f(1) = 0$, $f'(1) = 5$, $f'(2) = -4$, $g(1) = 2$, $g'(0) = -6$, and $g'(1) = 3$, then $F'(1) =$
    (a) -30
    (b) -24
    (c) -12
    (d) 0
    (e) 15

17. What can be said about the roots of the equation $x^3 + x + 6 = 0$?
    (a) it has no real roots
    (b) it has exactly 1 real root between -3 and -1
    (c) it has exactly 1 real root between -1 and 1
    (d) it has exactly 1 real root between 1 and 3
    (e) it has 3 real roots

18. $\int_{-1}^{1} x\sqrt{x^2+5}dx =$
    (a) $-\frac{4}{5}$
    (b) $-\frac{2}{5}$
    (c) 0
    (d) $\frac{1}{15}$
    (e) $\frac{2}{5}$

19. If $f(x) = \dfrac{1}{x^2}$ and $g(x) = \sqrt{x}$, then the domain of $f \circ g$ is
    (a) $(-\infty, \infty)$
    (b) $(0, \infty)$
    (c) $[0, \infty)$
    (d) $(-\infty, 0) \cup (0, \infty)$
    (e) None of the above

20. If $f(x) = \pi^3 + \dfrac{x}{\sqrt{x}}$, then $f'(x) =$
    (a) $3\pi^2 + \frac{1}{\frac{1}{2}x^{-1/2}}$
    (b) $3\pi^2 + \frac{1}{2\sqrt{x}}$
    (c) $\frac{1}{2\sqrt{x}}$
    (d) $\frac{\sqrt{x} - x\frac{1}{2}x^{-1/2}}{(\sqrt{x})^2}$
    (e) $3\pi^2 + \frac{\sqrt{x} - x\frac{1}{2}x^{-1/2}}{(\sqrt{x})^2}$

21. The domain of the function $f(x) = \sqrt{\dfrac{1-x}{1+x}}$ is the set of all real numbers x for which:
    (a) $x > 1$
    (b) $x \geq 1$
    (c) $-1 < x \leq 1$
    (d) $-1 < x$
    (e) $x \neq -1$

22. The graph of $y = x + \sin x$ has how many local maximums?
    (a) 0
    (b) 1
    (c) 2
    (d) 3

(e) infinitely many

23. A particle moves along a straight line with equation of motion $s(t) = \sqrt{t+1}$. Find its average velocity over the time interval $[0, 3]$.

(a) $\frac{1}{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) $\frac{14}{9}$

(d) 1

(e) $-\frac{1}{12}$

24. Evaluate $\lim_{x \to 7} \dfrac{\sqrt{x+2} - 3}{x - 7}$.

(a) 0

(b) $\frac{1}{6}$

(c) $\frac{1}{3}$

(d) 1

(e) ∞

25. Let $\mathcal{R}$ be the region enclosed by the lines $y = \sqrt{x}$ and $y = \dfrac{x}{2}$. The volume of the solid formed by rotating $\mathcal{R}$ about the x-axis is

(a) $2\pi \displaystyle\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx$

(b) $\pi \displaystyle\int_0^4 \left(\left(\frac{x}{2} - x\right)^2\right) dx$

(c) $\pi \displaystyle\int_0^4 \left(x - \left(\frac{x}{2}\right)^2\right) dx$

(d) $\pi \displaystyle\int_0^4 \left(\frac{x}{2} - \sqrt{x}\right)^2 dx$

(e) $2\pi \displaystyle\int_0^4 \left(\frac{x}{2} - \sqrt{x}\right)^2 dx$

Practice final: answer sheet

Turn in your answers by Dec 16. You will get credit only if you get more than 10 problems right. Your score will be the number of correct answers minus 10.

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