Math 75 **Fall 2003**

Practice test 2

The actual exam will consist of 6 multiple choice questions and 6 regular problems. You will have 1 hour to complete the exam.

Multiple choice questions: circle the correct answer

1. Find the derivative of $\sqrt{2x}$.

   A. $\frac{2}{\sqrt{x}}$ **B.** $\frac{2}{\sqrt{2x}}$ **C.** $\frac{1}{2\sqrt{x}}$ **D.** $\frac{1}{\sqrt{2x}}$ **E.** $\frac{1}{2\sqrt{2x}}$

2. Find the fifth derivative of $\cos(x)$.

   A. $\sin(x)$ **B.** $-\sin(x)$ **C.** $\cos(x)$ **D.** $-\cos(x)$ **E.** 0

3. Evaluate $\lim_{x\to-\infty} e^x$.

   A. $-\infty$ **B.** 0 **C.** 1 **D.** $+\infty$ **E.** does not exist

4. Find the horizontal asymptote of $f(x) = \frac{x+2}{x-5}$.

   A. $x = -2$ **B.** $y = -2$ **C.** $y = 1$ **D.** $x = 5$ **E.** $y = 5$

5. Find the vertical asymptote of $f(x) = \frac{x+2}{x-5}$.

   A. $x = -2$ **B.** $y = -2$ **C.** $y = 1$ **D.** $x = 5$ **E.** $y = 5$

Regular problems: show all your work

6. Differentiate the following functions:

   (a) $f(x) = 3\cos(x^5) + \frac{\pi}{2}$

   (b) $f(x) = \cos(4)(x^3 - 3x)$

   (c) $g(x) = \frac{x^3 - 5}{\cos(-x)}$

   (d) $h(x) = \tan(x)\left(\frac{1}{\sqrt[4]{x^3}} + \frac{2}{x}\right)$

7. Find the first five derivatives of $g(x) = 27x^{4/3}$

8. Find the points where the tangent line to the graph of $f(x) = x^5 - 80x$ is horizontal.

9. Find an equation of the tangent line to $y = \sqrt{2x+3}$ at $(3, 3)$.

10. Find the linearization of $g(x) = \sqrt{x}$ at $x = 1$ and use it to approximate $\sqrt{1.14}$.

11. Consider the curve given by $x^3y^3 - 3xy^3 + 4y = 6$.

    (a) Use implicit differentiation to find $y'(x)$.

    (b) Check that the point $(2, 1)$ lies on this curve.

    (c) What is the slope of the tangent line to this curve at $(2, 1)$?

12. A boy starts walking west at 6 km/h from a point P. Five minutes later a girl starts walking (a) north (b) east at 4 km/h from a point 15 km due south from P. At what rate is the distance between the kids changing 45 km after the girl starts walking? Is the distance increasing or decreasing at this instant?

13. A snowball is melting so that its radius is decreasing at a rate of 1 cm/min. Find the rate at which its volume is decreasing when the radius is 3 cm.

14. Find the critical numbers and local maxima and minima of $f(x) = x^3 - 3x^2 + 5$.

15. Find the absolute maximum and minimum values of $f(x) = \sin x$ on the interval $\left[0, \frac{5\pi}{4}\right]$.

16. Evaluate the limits:

    (a) $\lim_{x\to\infty} \dfrac{2x^3 + x - 5}{5x^3 - x^2}$

    (b) $\lim_{x\to-\infty} \dfrac{x + 1}{x^2 + 1}$

    (c) $\lim_{x\to\infty} \sqrt{x^2 + 3x - 2} - x$

    (d) $\lim_{x\to\infty} \tan x$

17. Let $f(x) = \dfrac{x}{(1 + x)^2}$. Find the following:

    (a) domain

    (b) intercepts

    (c) vertical and horizontal asymptotes

    (d) critical numbers

    (e) intervals of increase and decrease

    (f) local and absolute maxima and minima

    (g) intervals of concavity

    (h) inflection points

    (i) sketch the graph of $f(x)$