

**MATH 75**  
**Test 1 - Answers**

September 26, 2003

**Multiple choice questions: circle the correct answer**

1. Find domain of the function  $f(x) = \sqrt{x+1}$ .

- A.  $x > 0$       B.  $x \geq 0$       C.  $x \geq 1$       **D.  $x \geq -1$**       E.  $x \neq -1$

2. If  $f(x) = (x+1)^2$  and  $g(x) = 2x - 6$ , find  $f \circ g(x)$ .

- A.  $2x^2 + 4x - 4$       B.  $4x^2 + 8x - 5$       **C.  $4x^2 - 20x + 25$**       D.  $4x^2 - 24 + 36$

E. None of the above

3. Evaluate the limit:  $\lim_{x \rightarrow 5^+} \frac{|x-5|}{x-5}$ .

- A. 1**      B.  $-1$       C. 0      D.  $-\infty$       E. Does not exist

4. Find the derivative of  $3x^5$ .

- A.  $8x^4$       B.  $8x^5$       **C.  $15x^4$**       D.  $\frac{3(x+h)^5 - 3x^5}{h}$       E. Does not exist

5. Find the equation of the line tangent to the curve  $y = \frac{1}{x}$  at  $(2, \frac{1}{2})$ .

- A.  $y = -\frac{1}{2}x + \frac{3}{2}$       **B.  $y = -\frac{1}{4}x + 1$**       C.  $y = \frac{1}{2}x - \frac{1}{2}$       D.  $y = \frac{1}{2}x + \frac{1}{2}$

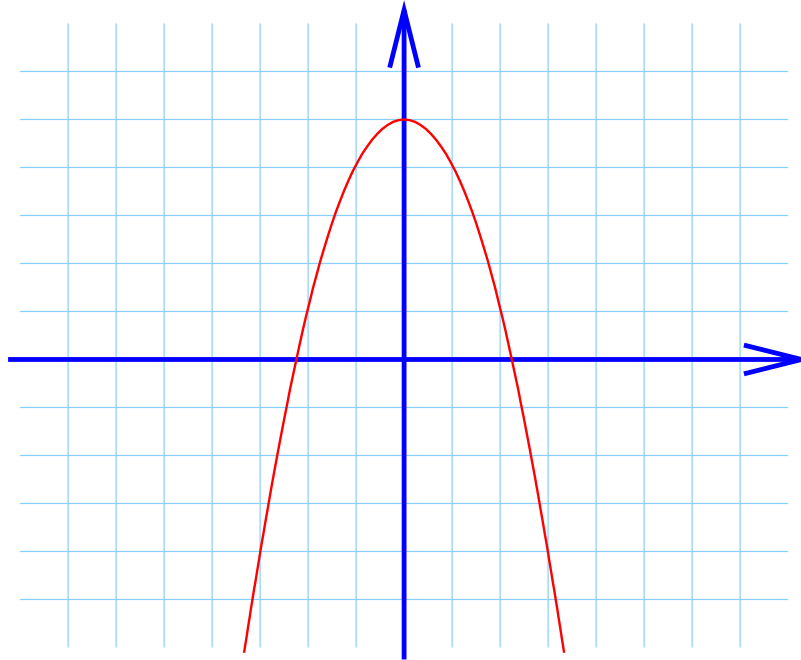
E. None of the above

6. If  $f(3) = 4$ ,  $f'(3) = 5$ ,  $g(3) = 6$ , and  $g'(3) = 7$ , find the derivative of the product  $f(x)g(x)$  at  $x = 3$ .

- A. 0      B. 12      C. 35      **D. 58**      E. Does not exist

**Regular problems: show all your work**

7. Sketch the graph of  $f(x) = 5 - x^2$ .



8. Show that the equation  $x^3 + 7x + 4 = 0$  has a root in the interval  $[-1, 1]$ .

*Let  $f(x) = x^3 + 7x + 4$ . Then  $f(-1) = -4 < 0$  and  $f(1) = 12 > 0$ . The function  $f(x)$  is continuous because it is a polynomial. By the intermediate value theorem, there is a number  $c$  between  $-1$  and  $1$  such that  $f(c) = 0$ .*

9. Evaluate the limit:  $\lim_{x \rightarrow 0} \frac{x^2 - x}{x^2 + x}$ . If the limit is infinite, determine whether it is  $+\infty$  or  $-\infty$ .

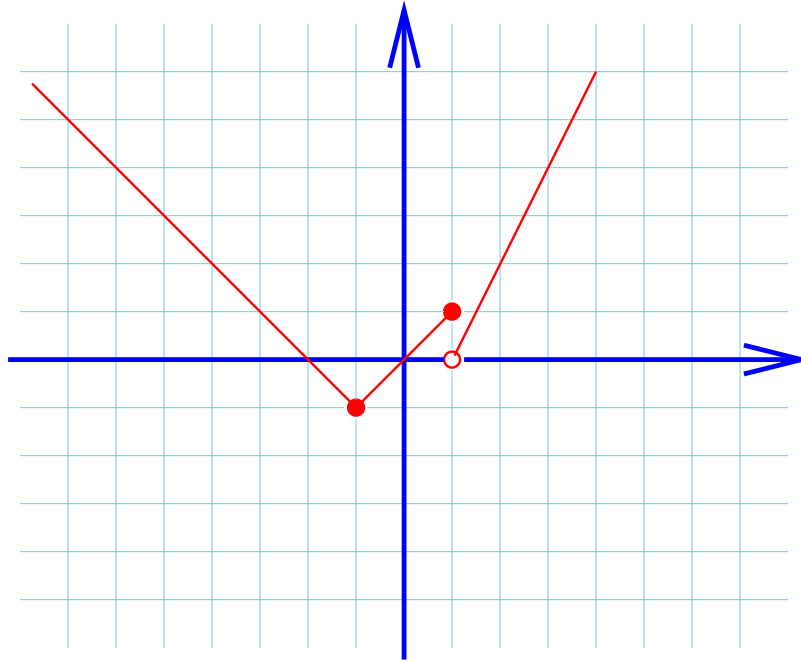
$$\lim_{x \rightarrow 0} \frac{x^2 - x}{x^2 + x} = \lim_{x \rightarrow 0} \frac{x(x - 1)}{x(x + 1)} = \lim_{x \rightarrow 0} \frac{x - 1}{x + 1} = \frac{-1}{1} = -1$$

10. Evaluate the limit:  $\lim_{x \rightarrow 5^+} \frac{x}{x^2 - 11x + 30}$ . If the limit is infinite, determine whether it is  $+\infty$  or  $-\infty$ .

$$\lim_{x \rightarrow 5^+} \frac{x}{x^2 - 11x + 30} = \lim_{x \rightarrow 5^+} \frac{x}{(x - 5)(x - 6)} \left[ \frac{\text{pos.}}{(\text{small pos.})(\text{neg.})} \right] = -\infty$$

11. Let  $f(x) = \begin{cases} -x - 2 & , \text{ if } x < -1 \\ x & , \text{ if } -1 \leq x \leq 1 \\ 2x - 2 & , \text{ if } x > 1 \end{cases}$ .

Sketch the graph of  $f(x)$ .



Is  $f(x)$  continuous at  $-1$ ?      *Yes*

Is  $f(x)$  continuous at  $1$ ?      *No*

12. Find the derivative of the function  $f(x) = \frac{x^5 - \sqrt{x}}{\frac{1}{x^2} + 4x^3}$ .

You do not have to simplify your derivative.

$$f'(x) = \frac{(5x^4 - \frac{1}{2}x^{-1/2})(\frac{1}{x^2} + 4x^3) - (x^5 - \sqrt{x})(-2x^{-3} + 12x^2)}{(\frac{1}{x^2} + 4x^3)^2}$$