

MATH 75
Test 1 - Answers

September 26, 2003

Multiple choice questions: circle the correct answer

1. Find domain of the function $f(x) = \sqrt{x+1}$.

- A. $x > 0$ B. $x \geq 0$ C. $x \geq 1$ **D. $x \geq -1$** E. $x \neq -1$

2. If $f(x) = (x+1)^2$ and $g(x) = 2x-6$, find $f \circ g(x)$.

- A. $2x^2 + 4x - 4$ B. $4x^2 + 8x - 5$ **C. $4x^2 - 20x + 25$** D. $4x^2 - 24 + 36$

E. None of the above

3. Evaluate the limit: $\lim_{x \rightarrow 5^+} \frac{|x-5|}{x-5}$.

- A. 1** B. -1 C. 0 D. $-\infty$ E. Does not exist

4. Find the derivative of $3x^5$.

- A. $8x^4$ B. $8x^5$ **C. $15x^4$** D. $\frac{3(x+h)^5 - 3x^5}{h}$ E. Does not exist

5. Find the equation of the line tangent to the curve $y = \frac{1}{x}$ at $(2, \frac{1}{2})$.

- A. $y = -\frac{1}{2}x + \frac{3}{2}$ **B. $y = -\frac{1}{4}x + 1$** C. $y = \frac{1}{2}x - \frac{1}{2}$ D. $y = \frac{1}{2}x + \frac{1}{2}$

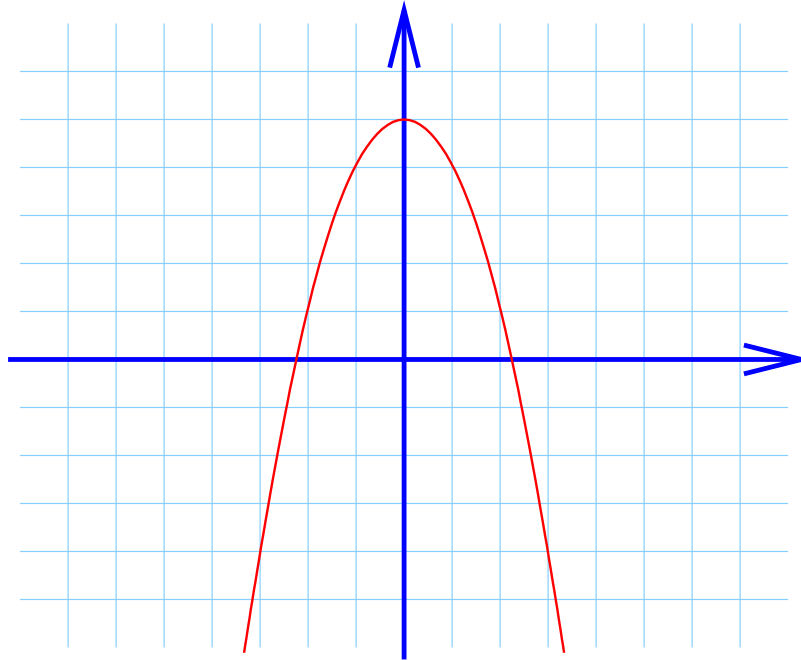
E. None of the above

6. If $f(3) = 4$, $f'(3) = 5$, $g(3) = 6$, and $g'(3) = 7$, find the derivative of the product $f(x)g(x)$ at $x = 3$.

- A. 0 B. 12 C. 35 **D. 58** E. Does not exist

Regular problems: show all your work

7. Sketch the graph of $f(x) = 5 - x^2$.



8. Show that the equation $x^3 + 7x + 4 = 0$ has a root in the interval $[-1, 1]$.

Let $f(x) = x^3 + 7x + 4$. Then $f(-1) = -4 < 0$ and $f(1) = 12 > 0$. The function $f(x)$ is continuous because it is a polynomial. By the intermediate value theorem, there is a number c between -1 and 1 such that $f(c) = 0$.

9. Evaluate the limit: $\lim_{x \rightarrow 0} \frac{x^2 - x}{x^2 + x}$. If the limit is infinite, determine whether it is $+\infty$ or $-\infty$.

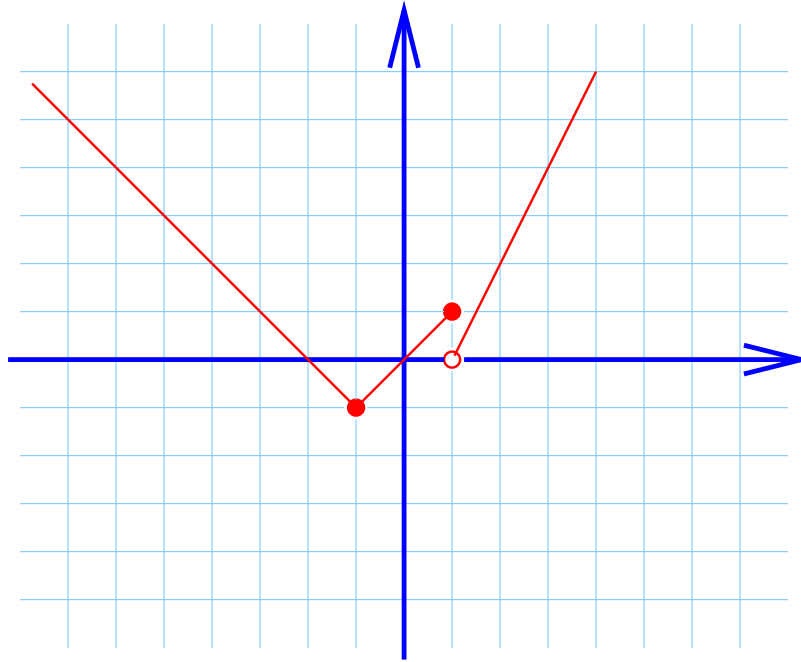
$$\lim_{x \rightarrow 0} \frac{x^2 - x}{x^2 + x} = \lim_{x \rightarrow 0} \frac{x(x - 1)}{x(x + 1)} = \lim_{x \rightarrow 0} \frac{x - 1}{x + 1} = \frac{-1}{1} = -1$$

10. Evaluate the limit: $\lim_{x \rightarrow 5^+} \frac{x}{x^2 - 11x + 30}$. If the limit is infinite, determine whether it is $+\infty$ or $-\infty$.

$$\lim_{x \rightarrow 5^+} \frac{x}{x^2 - 11x + 30} = \lim_{x \rightarrow 5^+} \frac{x}{(x - 5)(x - 6)} \left[\frac{\text{pos.}}{(\text{small pos.})(\text{neg.})} \right] = -\infty$$

11. Let $f(x) = \begin{cases} -x - 2 & , \text{ if } x < -1 \\ x & , \text{ if } -1 \leq x \leq 1 \\ 2x - 2 & , \text{ if } x > 1 \end{cases}$.

Sketch the graph of $f(x)$.



Is $f(x)$ continuous at -1 ? *Yes*

Is $f(x)$ continuous at 1 ? *No*

12. Find the derivative of the function $f(x) = \frac{x^5 - \sqrt{x}}{\frac{1}{x^2} + 4x^3}$.

You do not have to simplify your derivative.

$$f'(x) = \frac{(5x^4 - \frac{1}{2}x^{-1/2})(\frac{1}{x^2} + 4x^3) - (x^5 - \sqrt{x})(-2x^{-3} + 12x^2)}{(\frac{1}{x^2} + 4x^3)^2}$$