MATH 75

Test 2 - Solutions

Multiple choice questions: circle the correct answer

1. Find the derivative of the function $f(x) = \sqrt{x^2 - 1}$.

   A. $\dfrac{1}{2x\sqrt{x^2-1}}$ **B.** $\dfrac{1}{2\sqrt{x^2-1}}$ **C.** $\dfrac{1}{\sqrt{x^2-1}}$ **D.** $\dfrac{x}{\sqrt{x^2-1}}$ **E.** $2x\sqrt{x^2-1}$

2. Find the vertical and horizontal asymptotes for the function $f(x) = \dfrac{x}{x^2-1}$.

   A. $x = 1,\ x = -1$ **B.** $x = 1,\ y = 0$ **C.** $x = 0,\ y = 1,\ y = -1$ **D.** $x = 0,\ y = 1$

   E. $x = 1,\ x = -1,\ y = 0$

3. Evaluate the limit: $\lim\limits_{x\to\infty} \dfrac{9 - x^2}{5 + x}$.

   A. $-\infty$ **B.** -5 **C.** 0 **D.** 3 **E.** ∞

4. If $f(x) = \sin^2(x)$, find $f'\left(\frac{\pi}{4}\right)$.

   A. -2 **B.** -1 **C.** 0 **D.** $\dfrac{1}{2}$ **E.** 1

5. The graph of $y = 2x^3 - x^4$ has how many local maximums?

   A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

6. Find the inflection point(s) of the graph of $y = 2x^3 - x^4$.

   A. $(0, 0)$ only **B.** $(1, 1)$ only **C.** $\left(\frac{3}{2}, \frac{27}{16}\right)$ only **D.** $(0, 0)$ and $(1, 1)$

   E. $(0, 0)$ and $\left(\frac{3}{2}, \frac{27}{16}\right)$

Regular problems: show all your work

7. Evaluate the limit: $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{5x^2+4}}{3x+2}$

$$=\lim_{x\to-\infty}\frac{\frac{\sqrt{5x^2+4}}{x}}{\frac{3x+2}{x}}=\lim_{x\to-\infty}\frac{\frac{\sqrt{5x^2+4}}{-\sqrt{x^2}}}{3+\frac{2}{x}}=\lim_{x\to-\infty}\frac{-\sqrt{\frac{5x^2+4}{x^2}}}{3+\frac{2}{x}}=\lim_{x\to-\infty}\frac{-\sqrt{5+\frac{4}{x^2}}}{3+\frac{2}{x}}=-\frac{\sqrt{5}}{3}$$

8. Find the linear approximation of the function $f(x)=\dfrac{1}{x^2}$ at $a=-1$.

$L(x)=f(-1)+f'(-1)(x-(-1))$.

$f(-1)=\dfrac{1}{(-1)^2}=1.\qquad f'(x)=\dfrac{-2}{x^3}$, *so* $f'(-1)=\dfrac{-2}{(-1)^3}=2$.

Therefore the linear approximation is $L(x)=1+2(x+1)=2x+3$.

9. Find the intervals of increase and decrease of the function $f(x)=4x^3-3x^2-1$.

$f'(x)=12x^2-6x=6x(2x-1)$. *The derivative is* 0 *at* $x=0$ *and at* $x=\dfrac{1}{2}$.

Now consider each interval:

On the interval $(-\infty,0)$ *the derivative is positive, thus* $f(x)$ *is increasing.*

On the interval $\left(0,\dfrac{1}{2}\right)$ *the derivative is negative, thus* $f(x)$ *is decreasing.*

On the interval $\left(\dfrac{1}{2},+\infty\right)$ *the derivative is positive, thus* $f(x)$ *is increasing.*

10. The volume of a sphere of radius r is $V=\dfrac{4}{3}\pi r^3$. Air is pumped into a spherical balloon at the rate of 100 cm^3 per second. How fast is the radius of the balloon increasing when the radius is 10 cm?

Differentiating both sides of $V=\dfrac{4}{3}\pi r^2$ *with respect to* t*, remembering that both* V *and* r *are functions of* t *(time), gives* $V'=\dfrac{4}{3}\pi 3r^2r'$, *or* $V'=4\pi r^2 r$.

We are given that $V'=100$ *(air is pumped at the rate of 100* cm^3 *per second) and that* $r=10$ *(... when the radius is 10 cm), therefore we have*

$100=4\pi 10^2 r'$

$1=4\pi r'$

$r'=\dfrac{1}{4\pi}$

Thus the radius is increasing at a rate of $\dfrac{1}{4\pi}$ *cm per second.*

11. Find the absolute maximum and minimum values of $f(x) = (x^2 - 1)^3$ on the interval $[-1, 2]$.

First find the critical numbers. $f'(x) = 3(x^2 - 1)^2 2x = 6x(x^2 - 1)^2$. *The derivative is defined everwhere. Find the points where it is equal to 0:*

$6x(x^2 - 1)^2 = 0$

$6x((x - 1)(x + 1))^2 = 0$

$6x(x - 1)^2(x + 1)^2 = 0$

$x = 0$ *or* $x = 1$ *or* $x = -1$.

Thus 0, 1, *and* -1 *are critical values of* $f(x)$.

Now find the values of the function at the critical numbers and at the endpoints of the interval.

$f(0) = -1,\ f(1) = 0,\ f(-1) = 0,\ f(2) = 27$.

The largest of these values (27) is the absolute maximum value, and the smallest (−1) is the absolute minimum value.

12. Find an equation of the tangent line to the curve $xy^2 + 3xy = 4$ at the point $(1, 1)$.

First differentiate both sides of the equation with respect to x *treating* y *as a function of* x*:*

$y^2 + x2yy' + 3y + 3xy' = 0$

Now plug in $x = 1$ *and* $y = 1$*:*

$1 + 2y' + 3 + 3y' = 0$

$5y' = -4$

$y' = -\frac{4}{5}$

Thus the slope of the tangent line is $-\frac{4}{5}$. *The tangent line passes throught the point* $(1, 1)$, *so its equation is*

$y - 1 = -\frac{4}{5}(x - 1)$

$y = -\frac{4}{5}x + \frac{4}{5} + 1$

$y = -\frac{4}{5}x + \frac{9}{5}$