Math 75

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Practice test 1

The actual exam will consist of 6 multiple choice questions and 6 regular problems. You will have 1 hour to complete the exam.

**Multiple choice questions: circle the correct answer**

1. The function $f(x) = \sin(x) + x^2$ is
   A. even        B. odd        C. periodic with period $2\pi$        D. discontinuous at 0
   E. None of the above

2. If we shift the graph of $y = \sin(x)$ 2 units to the left, then the equation of the new graph is
   A. $y = \sin(x) + 2$        B. $y = \sin(x) - 2$        C. $y = \sin(x + 2)$        D. $y = \sin(x - 2)$
   E. $y = \sin(x/2)$

3. The domain of the function $f(x) = \sqrt{\frac{1}{9 - x^2}} + \sqrt{x-1}$ is the set of all real numbers $x$ for which
   A. $x < -3$ or $x > 3$        B. $x < 3$        C. $x \geq 1$        D. $1 \leq x < 3$        E. $1 < x < 3$

4. $\lim_{x \to -1^-} \frac{|x+1|}{x+1} =$
   A. 1        B. -1        C. 0        D. $-\infty$        E. Does not exist

5. The function $f(x) = \begin{cases} -x - 1 & \text{if } x < -1 \\ 0 & \text{if } -1 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases}$ is
   A. continuous everywhere
   B. continuous at 1 but discontinuous at $-1$
   C. continuous at $-1$ but discontinuous at 1
   D. continuous at all points except for 1 and $-1$
   E. discontinuous everywhere

6. Find the equation of the line tangent to the curve $y = x^2 + 4x + 4$ at (1, 9).
   A. $y = 9x$        B. $y = 6x - 15$        C. $y = 6x + 3$        D. $y = 2x + 1$
   E. None of the above

7. If $f(3) = 2$, $f'(3) = 4$, $g(3) = 5$, and $g'(3) = 6$, then the derivative of $\frac{f(x)}{g(x)}$ at $x = 3$ is
   $\left( \frac{f}{g} \right)'(3) =$
   A. 0.32        B. $2/3$        C. $-8/25$        D. 0        E. Undefined
Regular problems: show all your work

8. Sketch the graphs of:
   (a) \((x - 3)^2\)
   (b) \(3 \cos x + 2\)
   (c) \(-\sin (x - \frac{\pi}{2})\)
   (d) \(e^{-x-1}\)

9. Find a formula for the function whose graph is obtained from the graph of \(f(x) = e^x - 1\) by
   (a) Reflecting about the \(y\)-axis.
   (b) Vertically compressing by a factor of 5 and then shifting 3 units to the left.
   (c) Reflecting about the \(x\)-axis and then shifting 2 units down.

10. Let \(f(x) = 2 - x\), \(g(x) = \frac{1}{x}\), \(h(x) = \sqrt{x+1}\). Find the following functions and state their domains:
    (a) \(g \circ f\)
    (b) \(f \circ h\)
    (c) \(g \circ h\)

11. Evaluate the limits:
    (a) \(\lim_{x \to 5} (7x - 25)\)
    (b) \(\lim_{x \to -1} \frac{x^3 + x^2}{x^2 + 3x + 2}\)
    (c) \(\lim_{x \to 0} \frac{3 - \sqrt{9 + x}}{x}\)
    (d) \(\lim_{x \to 2^+} \frac{x^3 - 2}{x^2 - x - 2}\)
    (e) \(\lim_{x \to 2^-} \frac{x^3 - 2}{x^2 - x - 2}\)
    (f) \(\lim_{x \to 2} \frac{x^3 - 2}{x^2 - x - 2}\)
    (g) \(\lim_{x \to 0} x^4 \cos \left( \frac{1}{x} \right)\)

12. Find \(c\) such that the function \(f(x) = \begin{cases} cx & \text{if } x \geq 2 \\ 5 - x & \text{if } x < 2 \end{cases}\) is continuous everywhere.
13. Show that the equation $x^5 - 4x + 2 = 0$ has at least one solution in the interval $(1, 2)$.

14. Find the vertical asymptotes of $f(x) = \frac{(x + 2)(3x - 4)}{(x - 5)(x + 7)}$.

15. Differentiate the following functions:
   
   (a) $f(x) = 7x - 3$
   
   (b) $p(s) = s^5 - 2s^4 + 3s^3 - 4s^2 + 5s - 6$
   
   (c) $f(t) = \frac{3t^2 - 5t + 1}{\sqrt{t}}$
   
   (d) $g(x) = x^2 - \frac{x^3}{\sqrt{x}} + \frac{3}{x}$
   
   (e) $q(y) = \frac{y^2 + y + 1}{y + 1}$