MATH 75
Test 1 - Answers
February 23, 2005

Multiple choice questions: circle the correct answer

1. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x-5}}$.

   A. $x > 0$ **B.** $x \geq 0$ **C.** $x \neq 0$ **(D.)** $x > 5$ **E.** $x \neq 5$

2. If $f(x) = x^2$ and $g(x) = \cos x$, find $(f \circ g)(x)$.

   A. $x^2 \cos x$ **B.** $2x \cos x$ **(C.)** $\cos^2 x$ **D.** $\cos x^2$ **E.** None of these

3. Find the derivative of $x^2(x^3 + x)$.

   A. $2x(3x^2 + 1)$ **B.** $x^5 + x^3$ **(C.)** $5x^4 + 3x^2$
   D. $\dfrac{(x+h)^2((x+h)^3 + (x+h)) - x^2(x^3+x)}{h}$ **E.** Does not exist

4. Evaluate the limit: $\lim\limits_{x \to 4} \dfrac{x+4}{x-4}$

   A. 0 **B.** ∞ **C.** $-\infty$ **D.** 8 **(E.)** Does not exist

5. If $f(1) = 4$, $f'(1) = 3$, $g(1) = 2$, and $g'(1) = -1$, find the derivative of the quotient $\dfrac{f(x)}{g(x)}$ at $x = 1$.

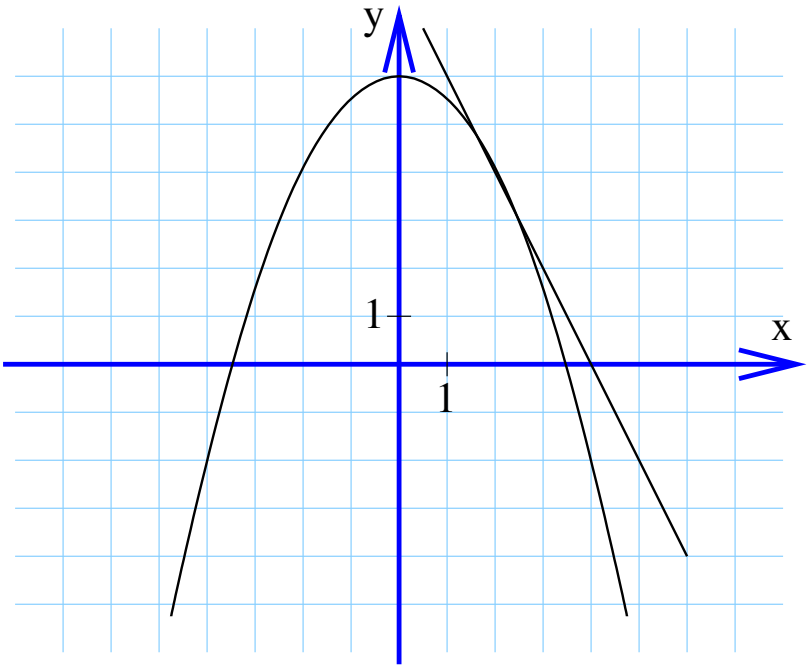
   A. -3 **B.** -2.5 **C.** 0.5 **(D.)** 2.5 **E.** 5

6. If the curve $y = \sin x$ is stretched vertically by a factor of 2 then the equation of the new curve is

   A. $y = \sin x + 2$ **B.** $y = \sin(x + 2)$ **(C.)** $y = 2 \sin x$ **D.** $y = \sin(2x)$ **E.** $2y = \sin x$

Regular problems: show all your work

7. Sketch the graph of $f(x) = 6 - \frac{1}{2}x^2$.



8. Find an equation of the tangent line to $y = 6 - \frac{1}{2}x^2$ at $(2, 4)$. Draw this tangent line on the above graph.

   The slope of the tangent line is $y'(2)$.
   $y'(x) = -\frac{1}{2}2x = -x$
   $y'(2) = -2$
   Then an equation is $y - 4 = -2(x - 2)$, *or* $y - 4 = -2x + 4$, *or* $y = -2x + 8$.

9. Show that the equation $x^5 + x - 5 = 0$ has a real root.

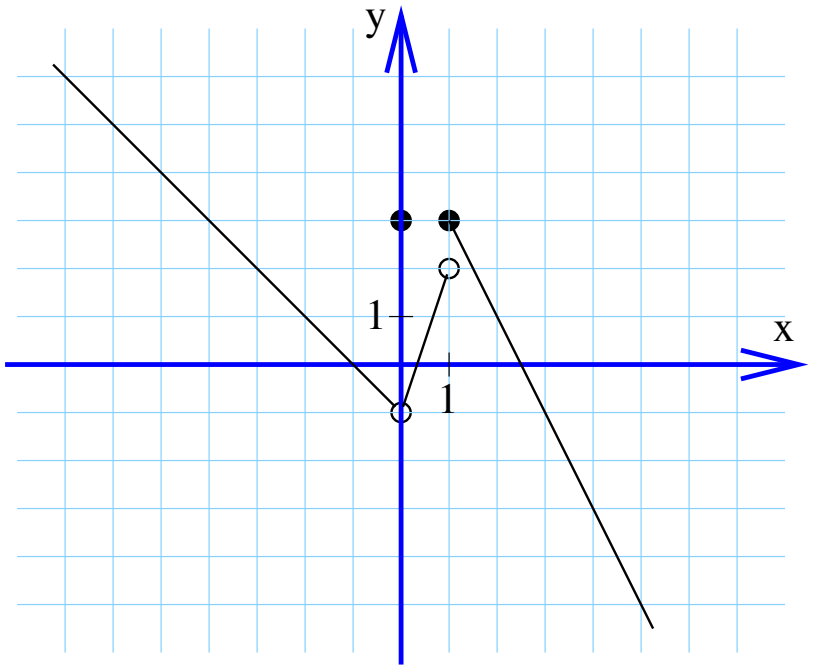
   Let $f(x) = x^5 + x - 5$, *then* $f(0) = -5 < 0$ *and* $f(2) = 29 > 0$, *therefore by the intermediate value theorem there exists a number* c *between* 0 *and* 2 *such that* $f(c) = 0$, *i.e.* $f(x)$ *has a real root.*

10. Evaluate the limit: $\lim_{x\to 4} \dfrac{\sqrt{x} - 2}{x - 4}$. If the limit is infinite, determine whether it is $+\infty$ or $-\infty$.

$$\lim_{x\to 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x\to 4} \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{(x - 4)(\sqrt{x} + 2)} = \lim_{x\to 4} \frac{(x - 4)}{(x - 4)(\sqrt{x} + 2)} = \lim_{x\to 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{4}$$

11. Let $f(x) = \begin{cases} -x-1 & , \text{ if } \quad x < 0 \\ 3 & , \text{ if } \quad x = 0 \\ 3x-1 & , \text{ if } \quad 0 < x < 1 \\ -2x+5 & , \text{ if } \quad x \geq 1 \end{cases}$.

Sketch the graph of $f(x)$.



Is $f(x)$ coninuous at 0? *No because* $\lim_{x \to 0} f(x) \neq f(0)$.

Is $f(x)$ continuous at 1? *No because* $\lim_{x \to 1} f(x)$ *does not exist.*

12. Find the derivative of the function $f(x) = x\sqrt{x}\left(5x - \dfrac{3}{x^4}\right)$.

Simplify your answer.

First rewrite the function: $f(x) = x^{\frac{3}{2}}\left(5x - 3x^{-4}\right) = 5x^{\frac{5}{2}} - 3x^{-\frac{5}{2}}$, *then*

$f'(x) = \dfrac{25}{2}x^{\frac{3}{2}} + \dfrac{15}{2}x^{-\frac{7}{2}} = \dfrac{25x\sqrt{x}}{2} + \dfrac{15}{2x^3\sqrt{x}}$