MATH 75
Test 2

April 4, 2005

Multiple choice questions: circle the correct answer

1. Find the derivative of $f(x) = \cos(2x^3)$.

   A. $\sin(2x^3)$ **B.** $\sin(6x^2)$ **C.** $-2x^3\sin(2x^3)$ **D.** $-6x^2\sin(2x^3)$ **E.** $-\sin(x)(6x^2)$

2. Find the vertical asymptotes of $f(x) = \dfrac{x^2}{x^2 - 3x}$.

   A. $x = 0$ **B.** $x = 3$ **C.** $x = 0$ and $x = 3$ **D.** $y = 1$ **E.** $y = 3$

3. Evaluate the limit: $\displaystyle\lim_{x\to-\infty} \frac{x^2+5}{x-3}$.

   A. 0 **B.** 1 **C.** $-\frac{5}{3}$ **D.** ∞ **E.** $-\infty$

4. If $f(t) = \dfrac{8}{x}$, find $f'''(2)$.

   A. -3 **B.** -2 **C.** 0 **D.** 1 **E.** 2

5. How many critical numbers does the function $y = x + \dfrac{1}{x}$ have?

   A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** infinitely many

6. Find the local maximum of $y = x + \dfrac{1}{x}$.

   A. $x = -2$ **B.** $x = -1$ **C.** $x = 0$ **D.** $x = 1$ **E.** $x = 2$

Regular problems: show all your work

7. Evaluate the limit: $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2+3x-4}}{5x-6}$.

$$\lim_{x\to\infty} \frac{\sqrt{x^2+3x-4}}{5x-6} = \lim_{x\to\infty} \frac{\frac{\sqrt{x^2+3x-4}}{x}}{5-\frac{6}{x}} = \lim_{x\to\infty} \frac{\frac{\sqrt{x^2+3x-4}}{\sqrt{x^2}}}{5-\frac{6}{x}} = \lim_{x\to\infty} \frac{\sqrt{1+\frac{3}{x}-\frac{4}{x^2}}}{5-\frac{6}{x}} = \frac{1}{5}$$

8. Find the linear approximation of the function $f(x) = x + \sin(x)$ at $a = 0$.

   The linear approximation is given by $L(x) = f(a) + f'(a)(x-a)$.
   Since $a = 0$, *we have* $L(x) = f(0) + f'(0)(x-0)$.
   $f(0) = 0$.
   $f'(x) = 1 + \cos(x)$.
   $f'(0) = 1 + 1 = 2$.
   So $L(x) = 0 + 2(x-0) = 2x$.

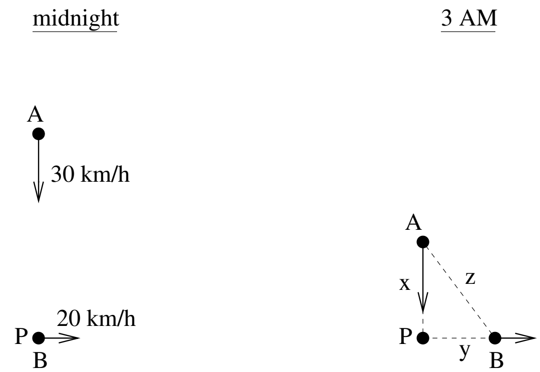
9. Find the intervals of increase and decrease of the function $f(x) = x^4 + 4x^3 + 5$.

   $f'(x) = 4x^3 + 12x^2$.
   The derivative is 0 when $4x^3 + 12x^2 = 0$, *i.e.* $4x^2(x+3) = 0$, *i.e. at* $x = 0$ *and* $x = -3$.
   It is easy to check that $f'(x) < 0$ *on* $(-\infty, -3)$, *and* $f'(x) > 0$ *on* $(-3, 0)$ *and on* $(0, +\infty)$.
   Therefore $f(x)$ *is increasing on* $(-3, +\infty)$ *and decreasing on* $(-\infty, -3)$.

10. Find the slope of the tangent line to the curve $x\tan y + xy + 3y = 0$ at the point $(0, 0)$.

    Rewrite the equation as $x\tan(y(x)) + xy(x) + 3y(x) = 0$.
    Differentiate with respect to x*:* $\tan(y(x)) + x\sec^2(y(x))y'(x) + y(x) + xy'(x) + 3y'(x) = 0$.
    Simplify: $\tan y + x\sec^2 y y' + y + xy' + 3y' = 0$.
    If $x = 0$ *and* $y = 0$, *we have* $0 + 0\cdot 1\cdot y' + 0 + 0\cdot y' + 3y' = 0$, *so* $y' = 0$.
    Thus the slope of the tangent line is 0.

11. At midnight, ship A is 170 km north of ship B. Ship A is sailing south at 30 km/h and ship B is sailing east at 20 km/h. How fast is the distance between the ships changing at 3:00 AM?



Let P be the starting point of ship B.
Let $x(t) = |AP|$, $y(t) = |PB|$, $z(t) = |AB|$ at time t.
We have $(x(t))^2 + (y(t))^2 = (z(t))^2$.
Differentiating with respect to t gives $2x(t)x'(t) + 2y(t)y'(t) = 2z(t)z'(t)$, or $2xx' + 2yy' = 2zz'$.
Divide both sides by 2: $xx' + yy' = zz'$.
It is given that $x' = -30$ and $y' = 20$.
At 3 AM (i.e. 3 hours after midnight), $x = 170 - 3 \cdot 30 = 80$, $y = 3 \cdot 20 = 60$, and $z = \sqrt{x^2 + y^2} = \sqrt{80^2 + 60^2} = \sqrt{6400 + 3600} = \sqrt{10000} = 100$.
So we have $80 \cdot (-30) + 60 \cdot 20 = 100z'$
$-2400 + 1200 = 100z'$
$z' = -12$
Thus at 3 AM the distance between the ships is decreasing at the rate of 12 km/h.

12. Find the absolute maximum and minimum values of $f(x) = x^4 + 4x^3 + 5$ on the interval $[-2, 0]$.

Use the closed interval method:
1. Find the critical numbers. We did this in problem 9, and got $x = 0$ and $x = -3$. However, -3 is not in our interval. Only 0 is.
2. Find the value of the function at the critical number(s): $f(0) = 5$.
3. Find the value of the function at the endpoints of the interval: $f(-2) = -11$. The other endpoint is 0, but we already found the value at 0.
4. The largest of the above values, i.e. 5, is the absolute maximum value of the function on the given interval, and the smallest of the above values, i.e. -11, is the absolute minimum value.