Math 75 **Maria Nogin**

Practice test 1 - Answers

The actual exam will consist of 6 multiple choice questions and 6 regular problems. You will have 1 hour to complete the exam.

Multiple choice questions: circle the correct answer

1. The function $f(x) = \sin(x) + x^2$ is
   A. even **B.** odd **C.** periodic with period 2π **D.** discontinuous at 0
   (**E.**) None of the above

2. If we shift the graph of $y = \sin(x)$ 2 units to the left, then the equation of the new graph is
   A. $y = \sin(x) + 2$ **B.** $y = \sin(x) - 2$ (**C.**) $y = \sin(x + 2)$ **D.** $y = \sin(x - 2)$
   E. $y = \sin(x/2)$

3. The domain of the function $f(x) = \sqrt{\dfrac{1}{9 - x^2}} + \sqrt{x - 1}$ is the set of all real numbers x for which
   A. $x < -3$ or $x > 3$ **B.** $x < 3$ **C.** $x \geq 1$ (**D.**) $1 \leq x < 3$ **E.** $1 < x < 3$

4. $\displaystyle\lim_{x \to -1^-} \frac{|x + 1|}{x + 1} =$
   A. 1 (**B.**) -1 **C.** 0 **D.** $-\infty$ **E.** Does not exist

5. The function $f(x) = \begin{cases} -x - 1 & \text{if } x < -1 \\ 0 & \text{if } -1 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases}$ is
   A. continuous everywhere
   B. continuous at 1 but discontinuous at -1
   (**C.**) continuous at -1 but discontinuous at 1
   D. continuous at all points except for 1 and -1
   E. discontinuous everywhere

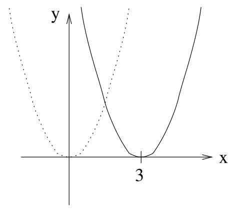
6. Find the equation of the line tangent to the curve $y = x^2 + 4x + 4$ at $(1, 9)$.
   A. $y = 9x$ **B.** $y = 6x - 15$ (**C.**) $y = 6x + 3$ **D.** $y = 2x + 1$
   E. None of the above

7. If $f(3) = 2$, $f'(3) = 4$, $g(3) = 5$, and $g'(3) = 6$, then the derivative of $\dfrac{f(x)}{g(x)}$ at $x = 3$ is $\left(\dfrac{f}{g}\right)'(3) =$
   (**A.**) 0.32 **B.** $2/3$ **C.** $-8/25$ **D.** 0 **E.** Undefined

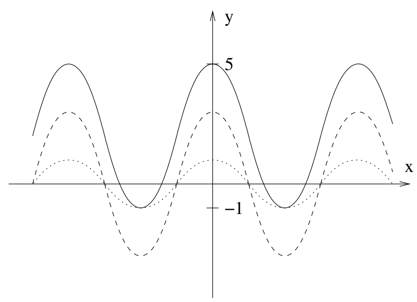
Regular problems: show all your work

8. Sketch the graphs of:

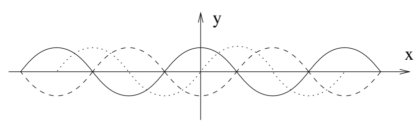
   (a) $(x-3)^2$: shift the graph of x^2 3 units to the right



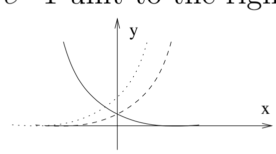
   (b) $3\cos x + 2$: stretch the graph of $\cos x$ by a factor of 3 vertically and then shift 2 units upward



   (c) $-\sin\left(x - \frac{\pi}{2}\right)$: shift the graph of $\sin x$ $\frac{\pi}{2}$ units to the right and then reflect about the x-axis



   (d) e^{-x-1}: shift the graph of e^x 1 unit to the right and then reflect about the y-axis



9. Find a formula for the function whose graph is obtained from the graph of $f(x) = e^x - 1$ by

   (a) Reflecting about the y-axis.
   $g(x) = e^{-x} - 1$

   (b) Vertically compressing by a factor of 5 and then shifting 3 units to the left.
   $$g(x) = \frac{e^{x+3} - 1}{5}$$

   (c) Reflecting about the x-axis and then shifting 2 units down.
   $g(x) = -(e^x - 1) - 2 = -e^x - 1$

10. Let $f(x) = 2 - x, \quad g(x) = \frac{1}{x}, \quad h(x) = \sqrt{x+1}$. Find the following functions and state their domains:

(a) $g \circ f = g(2-x) = \frac{1}{2-x}$, domain: $2 - x \neq 0$, so $x \neq 2$ or $(-\infty, 2) \cup (2, +\infty)$.

(b) $f \circ h = f(\sqrt{x+1}) = 2 - \sqrt{x+1}$, domain: $x + 1 \geq 0$, so $x \geq -1$ or $[-1, +\infty)$.

(c) $g \circ h = g(\sqrt{x+1}) = \frac{1}{\sqrt{x+1}}$, domain: $x + 1 > 0$, so $x > -1$ or $(-1, +\infty)$.

11. Evaluate the limits:

(a) $\lim_{x \to 5} (7x - 25) = 7 \cdot 5 - 25 = 10$

(b) $\lim_{x \to -1} \frac{x^3 + x^2}{x^2 + 3x + 2} = \lim_{x \to -1} \frac{x^2(x+1)}{(x+1)(x+2)} = \lim_{x \to -1} \frac{x^2}{x+2} = 1$

(c) $\lim_{x \to 0} \frac{3 - \sqrt{9+x}}{x} = \lim_{x \to 0} \frac{(3 - \sqrt{9+x})(3 + \sqrt{9+x})}{x(3 + \sqrt{9+x})} = \lim_{x \to 0} \frac{3^2 - (\sqrt{9+x})^2}{x(3 + \sqrt{9+x})} =$
$= \lim_{x \to 0} \frac{9 - (9+x)}{x(3 + \sqrt{9+x})} = \lim_{x \to 0} \frac{-x}{x(3 + \sqrt{9+x})} = \lim_{x \to 0} \frac{-1}{3 + \sqrt{9+x}} = -\frac{1}{6}$

(d) $\lim_{x \to 2^+} \frac{x^3 - 2}{x^2 - x - 2} = \lim_{x \to 2^+} \frac{x^3 - 2}{(x-2)(x+1)} \left[\frac{\text{pos.}}{(\text{small pos.})(\text{pos.})} \right] = +\infty$

(e) $\lim_{x \to 2^-} \frac{x^3 - 2}{x^2 - x - 2} = \lim_{x \to 2^+} \frac{x^3 - 2}{(x-2)(x+1)} \left[\frac{\text{pos.}}{(\text{small neg.})(\text{pos.})} \right] = -\infty$

(f) $\lim_{x \to 2} \frac{x^3 - 2}{x^2 - x - 2}$ DNE because the limits in (d) and (e) are not equal

(g) $\lim_{x \to 0} x^4 \cos\left(\frac{1}{x}\right) = 0$ by the squeeze theorem since $-x^4 \leq x^4 \cos\left(\frac{1}{x}\right) \leq x^4$ and $\lim_{x \to 0}(-x^4) = \lim_{x \to 0}(x^4) = 0$.

12. Find c such that the function $f(x) = \begin{cases} cx & \text{if} \quad x \geq 2 \\ 5 - x & \text{if} \quad x < 2 \end{cases}$ is continuous everywhere.

Since linear functions are continuous everywhere, $f(x)$ is continuous at all poits except possibly at 2. It is continuous at 2 if and only if the functions cx and $5 - x$ agree at 2 (that is, they have the same value at 2. The graph of $f(x)$ then has no jump at 2.) So we set the values of cx and $5 - x$ at 2 equal:
$c \cdot 2 = 5 - 2$
$2c = 3$
$c = \frac{3}{2}$

13. Show that the equation $x^5 - 4x + 2 = 0$ has at least one solution in the interval $(1, 2)$.

Let $f(x) = x^5 - 4x + 2$. Then $f(1) = -1 < 0$ and $f(2) = 26 > 0$. By the intermediate value theorem, there is a point c between 1 and 2 such that $f(c) = 0$.

14. Find the vertical asymptotes of $f(x) = \dfrac{(x+2)(3x-4)}{(x-5)(x+7)}$.

Since rational functions are continuous in their domains, $f(x)$ can have vertical asymptotes only at 5 and -7 (where it is undefined). Check the limits of $f(x)$ as x approaches 5 and -7:

$$\lim_{x\to 5^+} \frac{(x+2)(3x-4)}{(x-5)(x+7)} \left[\frac{(\text{pos.})(\text{pos.})}{(\text{small pos.})(\text{pos.})}\right] = +\infty$$

$$\lim_{x\to -7^+} \frac{(x+2)(3x-4)}{(x-5)(x+7)} \left[\frac{(\text{neg.})(\text{neg.})}{(\text{neg.})(\text{small pos.})}\right] = -\infty$$

Since the limits are infinite, $f(x)$ has vertical asymptotes $x = 5$ and $x = -7$.

15. Differentiate the following functions:

(a) $f(x) = 7x - 3$

$f'(x) = 7$

(b) $p(s) = s^5 - 2s^4 + 3s^3 - 4s^2 + 5s - 6$

$p'(s) = 5s^4 - 8s^3 + 9s^2 - 8s + 5$

(c) $f(t) = \dfrac{3t^2 - 5t + 1}{\sqrt{t}}$

$f(t) = 3t^{1.5} - 5t^{0.5} + t^{-0.5}$

$f'(t) = 4.5t^{0.5} - 2.5t^{-0.5} - 0.5t^{-1.5} = 4.5\sqrt{t} - \dfrac{2.5}{\sqrt{t}} - \dfrac{1}{2t^{1.5}}$

(d) $g(x) = x^2 - \dfrac{x^3}{\sqrt[4]{x}} + \dfrac{3}{x}$

$g(x) = x^2 - x^{11/4} + 3x^{-1}$

$g'(x) = 2x - \dfrac{11}{4}x^{7/4} - 3x^{-2}$

(e) $q(y) = \dfrac{y^2 + y + 1}{y + 1}$

$q'(y) = \dfrac{(2y+1)(y+1) - (y^2+y+1)(1)}{(y+1)^2} = \dfrac{y^2 + 2y}{(y+1)^2}$