Math 75 **Maria Voloshina**

Practice test 3

The actual exam will consist of 6 multiple choice questions and 6 regular problems. You will have 1 hour to complete the exam.

Multiple choice questions: circle the correct answer

1. $\int \cos(2x)dx =$

   A. $\sin(2x)+C$ **B.** $-\sin(2x)+C$ **C.** $\frac{1}{2}\sin(2x)+C$ **D.** $-2\sin(2x)$ **E.** $x^2\sin(2x)$

2. $\int_{-3}^{1}|x|dx =$

   A. -5 **B.** -4 **C.** 0 **D.** 4 **E.** 5

3. 600 square cm of material is available to make a box with an open top. The length and the height of the box have to be equal. Find the largest possible volume of such a box.

   A. 600 cm^3 **B.** $\frac{200^{3/2}}{3}$ cm^3 **C.** 1 L **D.** $\frac{4000}{3}$ cm^3 **E.** 2000 cm^3

4. Find the derivative of $f(x)=\int_2^x \sin(t^2)dt$

   A. $2t\cos(t^2)$ **B.** $\sin(2x)$ **C.** $\cos(x^2)$ **D.** $\sin(x^2)-\sin(4)$ **E.** $\sin(x^2)$

5. The area of the region enclosed by $y=\cos\left(\frac{\pi}{2}x\right)$ and $y=x^2-1$ is

   A. $\frac{12+4\pi}{3\pi}$ **B.** $\frac{4}{\pi+3}$ **C.** $\frac{10}{3}$ **D.** $\frac{14}{3}$ **E.** $\frac{4}{3}-\frac{4}{\pi}$

6. The region enclosed by $y=|x|-1$ and the x-axis is rotated about the x-axis. The volume of the obtained solid is

   A. $\pi\int_{-1}^{0}(-x+1)^2dx+\pi\int_{0}^{1}(x-1)^2dx$ **B.** $\pi\int_{-1}^{0}(x+1)^2dx+\pi\int_{0}^{1}(x-1)^2dx$

   C. $\pi\int_{-1}^{1}(x-1)^2dx$ **D.** $2\pi\int_{1}^{-1}(|x|-1)dx$ **E.** $2\pi\int_{-1}^{1}x(x-1)dx$

Regular problems: show all your work

7. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side 10 if one side of the rectangle lies on the base of the triangle.

8. Use Newton's method to approximate the root of the equation $x^2-23=0$. Choose a reasonable initial approximation x_1, and use it to find the second approximation x_2.

9. Find $f(x)$ if

   (a) $f'(x)=1-8x^3+2\sin x-\cos x,\ f(0)=5.$

   (b) $f''(x)=6-24x^2,\ f'(1)=-3,\ f(2)=-32.$

10. Evaluate the following integrals

    (a) $\int_1^3 (3x^2 - 6x + 5)dx$

    (b) $\int_\pi^{3\pi} \cos x dx$

    (c) $\int_{-1}^2 \sqrt{4 - t^2} dt$

    (d) $\int x \sin(x^2) dx$

    (e) $\int \frac{1}{(2 - 3s)^5} ds$

    (f) $\int \sin x \sqrt{\cos x} dx$

11. Find the derivative of $g(x) = \int_{3x}^{5x^2} \sqrt{t} \tan(3t) dt$

12. Estimate the value of $\int_0^{10} (x^2 + 6)dx$ using 5 approximating rectangles and

    (a) left endpoints,

    (b) right endpoints.

    (c) Evaluate $\int_0^{10} (x^2 + 6)dx$ using the Fundamental Theorem of Calculus.

    (d) Sketch the graph of $f(x) = x^2 + 6$ and explain the meaning of your answers in (a)-(d).

13. Sketch the region enclosed by $y = x^3 + 1$ and $y = 4x + 1$ and find its area.

14. Find the area of the region between the curves $x = y^2 - 3y$ and $x = 5y$ from $y = 0$ to $y = 3$.

15. Find the volume of the solid obtained by rotating the region under the curve $y = x^2$ from $x = 0$ to $x = 2$ about

    (a) the x-axis,

    (b) the y-axis,

    (c) the line $y = 4$,

    (d) the line $x = -1$.

16. Find the average value of $f(x) = \cos(\pi x/2)$ on the interval $[-1, 1]$.