MATH 75
Test 1 - Solutions
June 6, 2005

Multiple choice questions: circle the correct answer

1. Find the domain of the function $f(x) = \dfrac{5}{\sqrt{x}}$.

   A. $x < 0$ **B.** $x \leq 0$ **C.** $x \neq 0$ **D.** $x \geq 0$ **(E.)** $x > 0$

2. If $f(x) = \sin x$ and $g(x) = x^3$, find $(f \circ g)(x)$.

   A. $x^3 \sin x$ **B.** $3x^2 \cos x$ **C.** $\sin^3 x$ **(D.)** $\sin x^3$

   E. None of the above

3. Find the derivative of $\dfrac{x^3 + 1}{x^2}$.

   A. $\frac{3x^2}{2x}$ **B.** $\frac{3}{2}x$ **(C.)** $1 - \frac{2}{x^3}$ **D.** $\frac{5x^4+2x}{x^4}$ **E.** $\frac{2-x^3}{x^3}$

4. Evaluate the limit: $\lim_{x \to 4} \dfrac{x-2}{x+4}$

   A. 0 **B.** ∞ **C.** 1 **(D.)** $\frac{1}{4}$ **E.** Does not exist

5. If $f(0) = 1$, $f'(0) = 2$, $g(0) = 3$, and $g'(0) = 5$, find the derivative of the product $f(x)g(x)$ at $x = 0$.

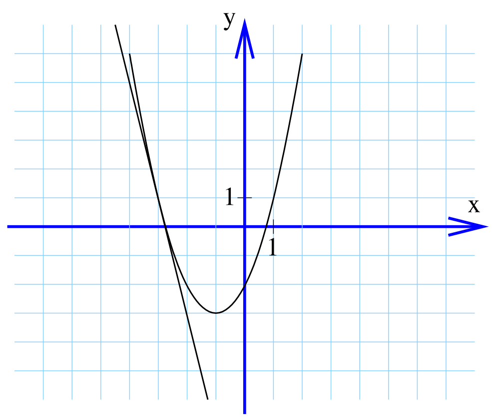
   A. -1 **B.** 0 **C.** 1 **D.** 10 **(E.)** 11

6. If the curve $y = \sin x$ is stretched horizontally by a factor of 2 then the equation of the new curve is

   A. $y = \sin x + 2$ **B.** $y = \sin(x + 2)$ **(C.)** $y = \sin\left(\frac{1}{2}x\right)$ **D.** $y = \sin(2x)$ **E.** $y = 2\sin x$

Regular problems: show all your work

7. Sketch the graph of $f(x) = (x + 1)^2 - 3$.



8. Find an equation of the tangent line to $y = (x+1)^2 - 3$ at $(-3, 1)$. Draw this tangent line on the above graph.

   $y = x^2 + 2x + 1 - 3 = x^2 + 2x - 2$
   $y' = 2x + 2$
   $y'(-3) = -4$, *so the slope of the tangent line is* -4.
   An equation of the tangent line is then $y - 1 = -4(x+3)$, *or*
   $y - 1 = -4x - 12$
   $y = -4x - 11$

9. Show that the equation $13x^5 + 5x + 13 = 0$ has a real root.

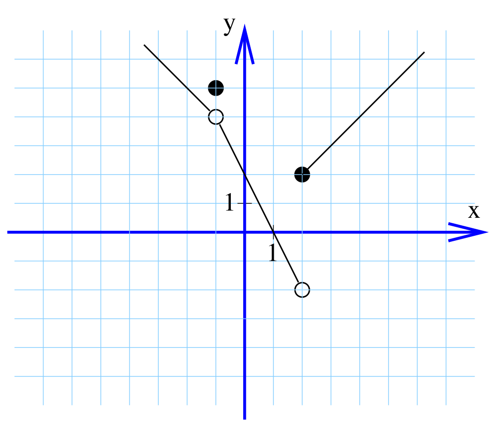
   Let $f(x) = 13x^5 + 5x + 13$. *Since* $f(x)$ *is a polynomial, it is continuous.*
   $f(0) = 13 > 0$, *and* $f(-1) = -5 < 0$, *therefore by the Intermediate Value Theorem* $f(x)$ *has a root in the interval (-1,0).*

10. Evaluate the limit: $\lim_{x\to 9} \dfrac{9-\sqrt{x}}{x-9}$. If the limit is infinite, determine whether it is $+\infty$ or $-\infty$.

    Since $\lim_{x\to 9^+} \dfrac{9-\sqrt{x}}{x-9} = +\infty$ *and* $\lim_{x\to 9^-} \dfrac{9-\sqrt{x}}{x-9} = -\infty$, $\lim_{x\to 9} \dfrac{9-\sqrt{x}}{x-9}$ *does not exist.*

11. Let $f(x) = \begin{cases} 3-x & , \text{ if } \quad x < -1 \\ 5 & , \text{ if } \quad x = -1 \\ -2x+2 & , \text{ if } \quad -1 < x < 2 \\ x & , \text{ if } \quad x \geq 2 \end{cases}$.

    Sketch the graph of $f(x)$.



    Is $f(x)$ coninuous at -1? *No because* $\lim_{x\to -1} f(x) \neq f(-1)$.

    Is $f(x)$ continuous at 2? *No because* $\lim_{x\to 2} f(x)$ *does not exist.*

12. Find the derivative of the function $f(x) = \dfrac{x^2}{\sqrt{x}}\left(5 + \dfrac{1}{x}\right)$. Simplify your answer.

    $f(x) = \dfrac{x^2}{\sqrt{x}}\left(5 + \dfrac{1}{x}\right) = x^{3/2}\left(5 + x^{-1}\right) = 5x^{3/2} + x^{1/2}$.

    $f'(x) = 5 \cdot \dfrac{3}{2}x^{1/2} + \dfrac{1}{2}x^{-1/2} = \dfrac{15\sqrt{x}}{2} + \dfrac{1}{2\sqrt{x}}$