MATH 75

Test 1

June 6, 2005

Name:__________________________________________

Email:__________________________________________

• No books, notes, or calculators are allowed.
• Please show all your work.
• Please simplify your answers.
Multiple choice questions: circle the correct answer

1. Find the domain of the function \( f(x) = \frac{5}{\sqrt{x}} \).
   A. \( x < 0 \)  
   B. \( x \leq 0 \)  
   C. \( x \neq 0 \)  
   D. \( x \geq 0 \)  
   E. \( x > 0 \)

2. If \( f(x) = \sin x \) and \( g(x) = x^3 \), find \((f \circ g)(x)\).
   A. \( x^3 \sin x \)  
   B. \( 3x^2 \cos x \)  
   C. \( \sin^3 x \)  
   D. \( \sin x^3 \)  
   E. None of the above

3. Find the derivative of \( \frac{x^3 + 1}{x^2} \).
   A. \( \frac{3x^2}{2x} \)  
   B. \( \frac{3}{2} x \)  
   C. \( 1 - \frac{2}{x^3} \)  
   D. \( \frac{5x^4 + 2x}{x^4} \)  
   E. \( \frac{2-x^3}{x^3} \)

4. Evaluate the limit: \( \lim_{x \to 4} \frac{x-2}{x+4} \)
   A. 0  
   B. \( \infty \)  
   C. 1  
   D. \( \frac{1}{4} \)  
   E. Does not exist

5. If \( f(0) = 1 \), \( f'(0) = 2 \), \( g(0) = 3 \), and \( g'(0) = 5 \), find the derivative of the product \( f(x)g(x) \) at \( x = 0 \).
   A. -1  
   B. 0  
   C. 1  
   D. 10  
   E. 11

6. If the curve \( y = \sin x \) is stretched horizontally by a factor of 2 then the equation of the new curve is
   A. \( y = \sin x + 2 \)  
   B. \( y = \sin(x + 2) \)  
   C. \( y = \sin \left( \frac{1}{2} x \right) \)  
   D. \( y = \sin(2x) \)  
   E. \( y = 2 \sin x \)
Regular problems: show all your work

7. Sketch the graph of \( f(x) = (x + 1)^2 - 3 \).

8. Find an equation of the tangent line to \( y = (x + 1)^2 - 3 \) at \((-3, 1)\). Draw this tangent line on the above graph.
9. Show that the equation $13x^5 + 5x + 13 = 0$ has a real root.

10. Evaluate the limit: $\lim_{x \to 9} \frac{9 - \sqrt{x}}{x - 9}$. If the limit is infinite, determine whether it is $+\infty$ or $-\infty$. 
11. Let \( f(x) = \begin{cases} 
3 - x & \text{if } x < -1 \\
5 & \text{if } x = -1 \\
-2x + 2 & \text{if } -1 < x < 2 \\
x & \text{if } x \geq 2 
\end{cases} \).

Sketch the graph of \( f(x) \).

Is \( f(x) \) continuous at \(-1\)?

Is \( f(x) \) continuous at \(2\)?

12. Find the derivative of the function \( f(x) = \frac{x^2}{\sqrt{x}} \left( 5 + \frac{1}{x} \right) \).

Simplify your answer.
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This page may be used as scratch paper