MATH 76

Test 2

October 29, 2004

Name:__________________________________________

• No books or calculators are allowed. One page of notes is allowed.
• Please show all your work.
• Please simplify your answers.
1. Estimate the value of the integral \( \int_{-3}^{3} \frac{1}{x^2 + 1} \, dx \) using the trapezoidal rule with \( n = 6 \).

2. Is the integral \( \int_{1}^{\infty} \frac{\ln x}{x} \, dx \) convergent or divergent? Explain why. If it is convergent, evaluate it.
3. Set up, but do not evaluate, an integral for the length of the curve $y = x^3$ from the point $(0, 0)$ to the point $(2, 8)$.

4. The curve $y = x^2$ between $x = 1$ and $x = 2$ is rotated about the $y$-axis. Find the area of the obtained surface.
5. (a) Solve the equation $x y y' = 1$.

(b) Find a solution of the above equation that satisfies the initial condition $y(1) = 2$. 
6. Sketch the curve $x = \sqrt{t} - 1, \ y = t - 2$ for $0 \leq t \leq 4$ and indicate with an arrow the direction in which the curve is traced as $t$ increases.
7. (a) Find polar coordinates of the point whose Cartesian coordinates are \((1, 1)\).

(b) Find Cartesian coordinates of the point whose polar coordinates are \(\left(2, \frac{\pi}{2}\right)\).

(c) Sketch the curve whose equation in polar coordinates is \(r = 2\).
Your total score in this class so far is out of .

Your current grade is .