MATH 76
Test 2 - Solutions
October 29, 2004

1. *Endpoints of subintervals are* $-3, -2, -1, 0, 1, 2, 3$. *The length of each subinteval is* 1. *The estimate is* $\frac{1}{2}\left(\frac{1}{(-3)^2+1}+\frac{2}{(-2)^2+1}+\frac{2}{(-1)^2+1}+\frac{2}{0^2+1}+\frac{2}{1^2+1}+\frac{2}{2^2+1}+\frac{1}{3^2+1}\right)=$ $\frac{1}{2}\left(\frac{1}{10}+\frac{2}{5}+1+2+1+\frac{2}{5}+\frac{1}{10}\right)=\frac{1}{2}\cdot 5=\frac{5}{2}$.

2. $\int_1^\infty \frac{\ln x}{x}dx = \lim_{t\to\infty}\int_1^t \frac{\ln x}{x}dx$ $\qquad u=\ln x,\ du=\frac{1}{x}dx.$

   $= \lim_{t\to\infty}\int_0^{\ln t} u du = \lim_{t\to\infty}\left.\frac{u^2}{2}\right|_0^{\ln t} = \lim_{t\to\infty}\left(\frac{(\ln t)^2}{2}-0\right)=\infty$, *so the integral diverges.*

3. $L=\int_0^2\sqrt{1+(3x^2)^2}dx=\int_0^2\sqrt{1+9x^4}dx$

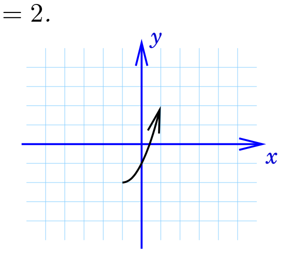
4. $S=2\pi\int_1^2 x\sqrt{1+(2x)^2}dx=2\pi\int_1^2 x\sqrt{1+4x^2}dx$ $\qquad u=1+4x^2,\ du=8xdx$

   $=\frac{2\pi}{8}\int_5^{17}\sqrt{u}du=\frac{\pi}{4}\left.\frac{2u^{3/2}}{3}\right|_5^{17}=\frac{2\pi}{4\cdot 3}\left(17^{3/2}-5^{3/2}\right)=\frac{\pi}{6}\left(17^{3/2}-5^{3/2}\right)$

5. (a) $xy\frac{dy}{dx}=1 \Rightarrow ydy=\frac{dx}{x} \Rightarrow \int ydy=\int\frac{dx}{x} \Rightarrow \frac{y^2}{2}=\ln|x|+C \Rightarrow y^2=2\ln|x|+K$ $\Rightarrow y=\pm\sqrt{2\ln|x|+K}$

   (b) *Plug in* 1 *for* x *and* 2 *for* y: $2=\pm\sqrt{0+K} \Rightarrow$ *have to use* $+$, *and* $K=4$. *So* $y=\sqrt{2\ln|x|+4}$.

6. *Solve* $x=\sqrt{t}-1$ *for* t: $\sqrt{t}=x+1 \Rightarrow t=(x+1)^2 \Rightarrow y=(x+1)^2-2$. *The graph of this equation is a parabola. Since we only want the curve for* $0\le t\le 4$, *the initial point has coordinates* $x=\sqrt{0}-1=-1$ *and* $y=0-2=-2$, *and the terminal point has coordinates* $x=\sqrt{4}-1=1$ *and* $y=4-2=2$.



7. (a) $r=\sqrt{1^2+1^2}=\sqrt{2}$, $\tan(\theta)=\frac{1}{1}$, $\theta=\frac{\pi}{4}$. *So polar coordinates are* $\left(\sqrt{2},\frac{\pi}{4}\right)$.

   (b) $x=2\cos\left(\frac{\pi}{2}\right)=0$, $y=2\sin\left(\frac{\pi}{2}\right)=2$. *So Cartesian coordinates are* $(0,2)$.

   (c) *The curve* $r=2$ *consists of all points with distance to the origin equal to 2.*