Math 76 **Practice problems for test 3**

1. Find the area of the region enclosed by one loop of $r = \sin(4\theta)$.

2. Find the length of the curve given by $r = 5\cos\theta$, $0 \le \theta \le \dfrac{3\pi}{4}$.

3. Find an equation and sketch the graph of the parabola with focus at $(1, -1)$ and directrix $y = 5$.

4. Find the vertices, foci, and asymptotes of the hyperbola give by $9x^2 - y^2 = 9$ and sketch its graph.

5. Find the vertices and foci of the ellipse given by $9x^2 - 18x + 4y^2 = 27$ and sketch its graph.

6. Determine whether the sequence converges or diverges. If it converges, find the limit.
   (a) $a_n = \dfrac{\sqrt{n}}{1+\sqrt{n}}$ (b) $a_n = 2 + \cos(n\pi)$

7. Determine whether the series is convergent or divergent. Explain your reason. If the series is convergent, find its sum.
   (a) $\displaystyle\sum_{n=1}^{\infty} \arctan n$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{5}}{3^n}$

8. Determine whether the series is convergent or divergent. Explain your reason.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n!}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - 5n}{n^3 + n - 1}$
   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln^2((n+1)^3)}$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{3^{1+3n}}$

9. Find the radius of convergence and the interval of convergence of the series.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

10. Find a power series representation for $\dfrac{x}{4x+1}$ and determine the interval of convergence.

11. Evaluate the integral $\displaystyle\int \frac{1}{1+x^4}\,dx$ as a power series.

12. Find the Taylor series for $f(x) = \dfrac{1}{x}$ at $a = 1$.