CSU FRESNO MATH PROBLEM SOLVING

March 21, 2009 Problems

Topic 1: Equations and inequalities with radicals, exponents, and logs

1. (MH 9-10 2002) Solve: $2^x = \frac{1}{64}$
   (a) $x = 6$
   (b) $x = -6$
   (c) $x = 4$
   (d) none of the above
2. (MH 11-12 2000) Solve for x: $3^{\log_3(8x-4)} = 5$
   (a) $\frac{9}{8}$
   (b) $\frac{9}{4}$
   (c) $\frac{8}{5}$
   (d) $\frac{8}{9}$
   (e) None of the above
3. (MH 9-10 2005) Solve for x: $\sqrt{1+\sqrt{2+\sqrt{x}}} = 3$.
   (a) 78
   (b) 3844
   (c) 15
   (d) none of the above
4. (MH 11-12 2005) How many real solutions are there to the equation $\sqrt{x^2+1} + \sqrt{x} = 1$?
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4
5. (MH 11-12 2003) How many roots does the equation $\sqrt{x^2+1} + \sqrt{x^2+2} = 2$ have?
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) None of the above
6. (MH 9-10 1998) Solve for x: $(6^{x+3} \cdot 6^{2x-1}) = 1$.
   (a) $\frac{2}{3}$

(b) $\frac{3}{2}$

(c) $\frac{-2}{3}$

(d) none of the above

7. (MH 9-10 2005) Solve for x: $4^x - 4^{x-1} = 12$.

   (a) 2

   (b) 3

   (c) 9

   (d) none of the above

8. (MH 11-12 2008) Solve for x: $9^x - 4 \cdot 3^{x+1} + 27 = 0$

   (a) $x = 3$ and $x = 9$

   (b) $x = -1$ and $x = -2$

   (c) $x = 1$ and $x = 2$

   (d) $x = -3$ and $x = -9$

9. (LF 9-12 2000) The real solution to the equation $\frac{81^{x+2}}{9^{3x+4}} = 9^{5x+1}$ is $x =$

   (a) $\frac{-1}{3}$

   (b) $\frac{-2}{3}$

   (c) $\frac{-3}{4}$

   (d) $\frac{-1}{6}$

   (e) None of these

10. (MH 11-12 2005) Solve for x: $3(8^x) + 9(4^x) - 30(2^x) = 0$.

    (a) 0

    (b) 1

    (c) 2

    (d) -5

    (e) There is no solution

11. (MH 11-12 2003) Given that $9^x + 9^{-x} = 34$, find $3^x + 3^{-x}$.

    (a) 3

    (b) 6

    (c) 9

    (d) 27

    (e) 81

12. (MH 11-12 1997) If $5^{3\log_5 x} = 64$, then

    (a) $x = 5$

    (b) $x = 125$

    (c) $x = \frac{64}{3}$

(d) $x = 4$

(e) None of the above

13. (MH 11-12 2005) Find the value of n if $\log_2(\log_5(\log_4 2^n)) = 2$.

(a) 0

(b) 4

(c) 25

(d) 625

(e) 1,250

14. (MH 11-12 2003) Find the natural n such that $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \ldots \cdot \log_n(n+1) = 10$

(a) 9

(b) 10

(c) 100

(d) 1023

(e) Does not exist

15. (MH 9-10 1998) Solve for x: $\log_{10}(x^2+3x) + \log_{10}(5x) = 1 + \log_{10}(2x)$.

(a) 10

(b) 1

(c) -5

(d) $\frac{1}{5}$

16. (MH 11-12 2006) Solve for x: $\log_2 x + \log_3 x = 3 + \log_2 3 + \log_3 4$

(a) $\frac{1}{6}$

(b) $\frac{2}{3}$

(c) $\frac{3}{2}$

(d) 6

(e) 12

17. (MH 11-12 2005) Solve $x - xe^{3x-8} = 0$.

(a) $x = 0$

(b) $x = \frac{8}{3}$

(c) $x = -\frac{8}{3}$

(d) $x = \frac{3}{8}$

(e) $x = 0$ and $x = \frac{8}{3}$

18. (MH 11-12 2003) Solve for x: $\sqrt{x^2 - x - 12} < x$

(a) $x \in (-12, +\infty)$

(b) $x \in [4, +\infty)$

(c) $x \in (12, +\infty)$

(d) No solutions exist

(e) None of the above

19. (MH 11-12 2003) Solve for x: $\log_{x^2-3} 729 > 3$

(a) $x \in (0, +\infty)$

(b) $x \in (-\sqrt{12}, -2)$

(c) $x \in (3, +\infty)$

(d) $x \in (2, \sqrt{12})$

(e) (b) or (d)

Topic 2: Complex numbers

Simplifying/evaluating expressions involving complex numbers

1. (MH 11-12 2005) Divide $\frac{3-2i}{2+4i}$.

(a) $-\frac{1}{10} - \frac{2}{5}i$

(b) $-\frac{1}{10} - \frac{4}{5}i$

(c) $\frac{7}{10} + \frac{2}{5}i$

(d) $\frac{4}{5} - \frac{1}{10}i$

(e) None of the above

2. (MH 11-12 2005) If i is the imaginary number, what is i^{85}?

(a) 1

(b) -1

(c) i

(d) $-i$

(e) None of the above

3. (MH 9-10 2005) Simplify $(1-(-i)^{318})^2$.

(a) 4

(b) i

(c) 0

(d) none of the above

4. (MH 11-12 2005) Determine the real part of $(1+2i)^5$.

(a) 1

(b) 41

(c) 17

(d) 121

(e) None of the above.

5. (MH 9-10 2002) Find: $\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^3$

(a) i

(b) $-i$

(c) -1

(d) 1

6. (MH 11-12 2005) Determine the polar representation of $(\sqrt{3}-i)^4$.

   (a) $16e^{i\frac{2\pi}{3}}$

   (b) $16e^{i\frac{4\pi}{3}}$

   (c) $16e^{i\frac{5\pi}{3}}$

   (d) 16

   (e) None of the above

7. (MH 11-12 2000) Simplify: $\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^{10}$

   (a) i

   (b) $-i$

   (c) 1

   (d) -1

   (e) None of the above

8. (MH 11-12 2005) Determine the polar representation of $\frac{2-2i}{1+i}$.

   (a) $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

   (b) $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

   (c) $2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

   (d) $2\left(\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}\right)$

   (e) None of the above

9. (MH 11-12 2000) Convert to polar notation and multiply: $(1+i)(\sqrt{3}-i)$

   (a) $2\sqrt{3}(\cos 60^0+i\sin 60^0)$

   (b) $2\sqrt{2}(\cos 15^0+i\sin 15^0)$

   (c) $2\sqrt{3}(\cos 30^0+i\sin 30^0)$

   (d) $2\sqrt{2}(\cos 45^0+i\sin 45^0)$

   (e) None of the above

10. (LF 9-12 2000) Suppose $z=1-i$. The real part of $1+z+z^2+z^3+\ldots+z^{99}$ is

    (a) 0

    (b) -1

    (c) $-1-2^{50}$

    (d) $-2^{49}\sqrt{2}$

    (e) None of these

11. (MH 11-12 2005) Let $z=x+iy$. Determine the real part of $z^2/\bar{z}$.

(a) $\frac{x^2-y^2}{x^2+y^2}$

(b) $\frac{3x^2y+y^3}{x^2+y^2}$

(c) $\frac{3x^2y-y^3}{x^2+y^2}$

(d) $\frac{x^3+3xy^2}{x^2+y^2}$

(e) $\frac{x^3-3xy^2}{x^2+y^2}$

12. (LF 9-12 2002) Suppose w and z are two complex numbers that satisfy $wz = 1$ and $w + z = -1$. Then $w^{16} + z^{16} =$

(a) i

(b) 1

(c) -1

(d) $-i$

(e) None of these

Roots of polynomials

13. (MH 11-12 1997) If a polynomial with real coefficients has $2+i\sqrt{5}$ and 6 as roots, then another root of the polynomial is:

(a) $-2+i\sqrt{5}$

(b) $6i$

(c) $-2-i\sqrt{5}$

(d) There need not be another root.

(e) There is another root but it is none of the above.

14. (MH 11-12 2005) How many roots does the polynomial $z^3 + 64$ have?

(a) No roots

(b) One real repeated root

(c) Two real roots, one of which is repeated

(d) Two real roots and one complex root

(e) One real root and a pair of complex conjugate roots

15. (MH 11-12 2000) What is the polynomial of lowest degree with rational coefficients that has $2+\sqrt{3}$ and $1-i$ as some of its roots?

(a) $x^4 - 8x^3 + 12x^2 - 10x + 2$

(b) $x^4 - 6x^3 + 11x^2 - 10x + 2$

(c) $x^4 + 6x^3 + 11x^2 + 10x + 4$

(d) $x^4 - 12x^3 + 11x^2 - 10x + 12$

(e) None of the above

16. (LF 9-12 1998) The sum of the four distinct complex roots to the polynomial $x^4 + 2x^3 + 3x^2 + 4x + 5$ is

(a) 4

(b) $\sqrt{5}$

(c) i

(d) $4i$

(e) None of these

See solutions at `http://zimmer.csufresno.edu/~mnogin/mfd-prep.html`