Department Qualifying Exam (*Traditional M.A. Students*)
Algebra Syllabus

Note: The exam consists of two sections (abstract algebra and linear algebra) with 8 questions per sections. Students must answer 5 questions per section. The exam topics are normally covered in Math 151 and Math 152.

Topics: The student is expected to know at least the following topics. Although the list is reasonably comprehensive, it is just an indication of all topics that could be covered in the exam. It is the student's responsibility to prepare adequately for the exam.

Part I (Abstract Algebra):

1. equivalence relations;
2. basic properties of the *gcd* and *lcm* of two integers;
3. groups: both additive and multiplicative, Abelian and non-Abelian. Must know how to check a set (with an operation) is a group, examples and properties of well-known groups ($\mathbb{Z}_n$, D_n, S_n, A_n, Q_8, etc);
4. the order of a group, and of an element in a group; Lagrange's theorem, Euler's theorem,
5. cyclic groups;
6. permutation groups (S_n and A_n); multiplication, even and odd permutations;
7. subgroups: examples of well-known subgroups, checking for a set to be a subgroup of a given group, cosets of a subgroup, index of a subgroup;
8. normal subgroups and factor (quotient) groups;
9. group homomorphisms; kernel and image of a group homomorphism, isomorphisms;
10. first isomorphism theorem and corollaries (the other homomorphism theorems);
11. direct product/sum of groups;
12. rings: must know how to check a set, with two given operations, is a ring (note that for this exam all rings have a unity), examples and properties of well-known rings ($\mathbb{Z}$, $\mathbb{Z}_n$, polynomial rings, matrices, etc), invertible elements, zero-divisors;
13. $\mathbb{Z}$ and $\mathbb{Z}_n$, Chinese remainder theorem;
14. subrings; left, right and two-sided ideals;
15. ring homomorphisms; lernel and image of a ring homomorphism, isomorphisms;
16. basic properties of integral domains, division rings and fields.

Part II (Linear Algebra):

1. vector spaces over $\mathbb{R}$ and $\mathbb{C}$; know how to work with the basic examples of vector spaces: $\mathbb{R}^n$, $\mathbb{C}^n$, matrices, $\mathcal{P}_n$, function spaces, etc.;
2. spanning sets, linearly independent sets, bases; change of basis; dimension;
3. linear transformations; must be able to find a matrix for a linear map in any given pair of bases;

4. kernel and range of a linear map; nullity and rank; dimension formula;
5. matrices: symmetric, diagonal, elementary, echelon form, upper/lower diagonal, block matrices, etc.; matrix algebra: inverses, transposes, etc.;
6. determinants; relation with the invertibility of a matrix;
7. how to solve systems of equations using elementary operations; know how homogeneous systems of equations yield subspaces;
8. eigenvalues (for this exam only real eigenvalues will be considered), eigenvectors, eigenspaces;
9. diagonalization of matrices, including with (maybe) repeated eigenvalues;
10. inner product spaces; Gram-Schmidt orthonormalization process.

Suggested References:

- *Contemporary Abstract Algebra* by Joseph A. Gallian
- *A First Course in Abstract Algebra* by John B. Fraleigh
- *Abstract Algebra* by I.N. Herstein
- *Abstract Algebra* by John Beachy and William Blair
- *A First Course in Abstract Algebra* by Joseph Rotman
- *Linear Algebra* by K. Hoffman and R. Kunze
- *Elementary Linear Algebra* by Ron Larson and David Falvo
- *Linear Algebra* by Steven J. Leon.
- *Elementary Linear Algebra* by David Kolman and Bernard Hill.