Marx's semantics and the logic of the derivation of value

One of the oldest and yet still unresolved controversies in the history of economic theory centers around the first few pages of Capital, where Marx derives his concept of value. This controversy began with Bohm-Bawerk's famous work, Karl Marx and The Close of His System. In this work Bohm-Bawerk praised the middle parts of the Marxian system as "masterly." These parts, he argued, "by their extraordinary logical consistency, permanently establish the reputation of the author as an intellectual force of the first rank" (Bohm-Bawerk 1949, 88-89). However, he contended that the beginning of Marx's system, where he derives the concept of value, and the end of his system, where he transforms market values into prices of production, are logically flawed. In these two "most decisive places," Bohm-Bawerk argued, Marx resorted to sophistry in order to achieve his desired results (90). The beginning was "a subtle and artificial afterthought contrived to make a preconceived opinion seem a natural outcome of a prolonged investigation" (69). Marx, said Bohm-Bawerk, "knew the result that he wished to obtain, and must obtain, and so he twisted and manipulated the long-suffering ideas and logical premises with admirable skill and subtlety until they actually yielded the desired result in a seemingly respectable syllogistic form" (79). The end, contended Bohm-Bawerk, was written by Marx when "he could not venture to write clearly and definitely without open contradiction and retraction" (100). This explained, according to Bohm-Bawerk, Marx's logical blunders that appear at the beginning and the end.

A few years later Hilferding wrote a counter-critique in which he argued that Bohm-Bawerk's critique of Marx's Capital represented the clash of two "mutually exclusive outlooks upon the whole of social life" (Hilferding 1949, 187). This clash, according to Hilferding, had created for Bohm-Bawerk a perception of contradictions in Marx's theory where none existed. Hilferding applied this general line of defense against Bohm-Bawerk's argument that there was an open contradiction between the two volumes of Capital. He showed that this contradiction existed not in Marx's theory but in Bohm-Bawerk's interpretation of the theory. This argument was coupled with pointing out a major factual error in Bohm-Bawerk's contention. The third volume of Capital, he reminded Bohm-Bawerk, was written before the publication of the first volume and thus could not have been a cunning postscript (155). The debate over the apparent contradiction between the two volumes came to an end. However, Hilferding was not as successful in defending the logical consistency of the beginning of Capital. He failed to address the important charge made by Bohm-Bawerk that the beginning of Capital was an artificial afterthought. He also failed to convince many future critics that Bohm-Bawerk's perception of logical blunder in the derivation of value was due to his interpretation of Marx's method and economic concepts. This left the controversy on the beginning of Capital unresolved. Since then, every few years, the debate over Marx's derivation of value erupts. In this periodic debate, the critics argue, a la Bohm-Bawerk, that this derivation is logically flawed and the counter-critics argue otherwise, mainly by quoting Marx and repeating what has been said before. So far, there seems to be no resolution in sight.

Towards resolving this century-old controversy, I would like to make the following points: Bohm-
Bawerk's assertion that the beginning of *Capital* was merely "an artificial afterthought" was factually incorrect and the derivation of value, contrary to his claim, was actually the result of a prolonged amount of work by Marx. Thus, any logical error in the derivation, perceived or actual, could not have been the result of sophistry on Marx's part. This is demonstrated in the second section of this article. Moreover, both Bohm-Bawerk's criticism of Marx's derivation and the later counter-criticisms of Bohm-Bawerk's work by Marxist writers are for the most part irrelevant because neither side has correctly understood the thrust of Marx's argument. This issue is discussed in the fourth and the last section of the present paper. However, I will argue in the third section that Marx himself is at least partially responsible for the misunderstandings that may arise, and have arisen, on the part of the reader because some of the terms that he employs, such as "use-value," are not clearly defined. Furthermore, ill-defined or fuzzy concepts used by Marx, such as "exchange-value" and the "exchange relation," result in a fallacious argument at the most decisive point in the derivation. This fallacy, so far as I know, has never been pointed out in any previous criticism of Marx's derivation. But before dealing with these issues in detail, let me first outline the main points of the controversy.

**I. Marx's derivation and Bohm-Bawerk's critique**

The focus of the controversy has usually been on a few consecutive paragraphs that appear in the first few pages of either the third German edition or the English edition of the first volume of *Capital*. Given the importance of these paragraphs to the previous debates and to my own attempt to analyze them in the third section of this article, I will reproduce them in their entirety, numbering them for convenience.

1. A given commodity, a quarter of wheat for example, is exchanged for x boot-polish, y silk or z gold, etc. In short, it is exchanged for other commodities in the most diverse proportion. Therefore the wheat has many exchange-values instead of one. But x boot-polish, y silk, z gold, etc., each represent the exchange-value of one quarter of wheat. Therefore x boot-polish, y silk, z gold, etc., must, as exchange-values, be mutually replaceable or of identical magnitude. It follows from this that, firstly, the valid exchange-values of a particular commodity express something equal, and secondly, exchange-value cannot be any thing other than the mode of expression, the 'form of appearance', of a content distinguishable from it.

2. Let us now take two commodities, for example corn and iron. Whatever their exchange relation may be, it can always be presented by an equation in which a given quantity of corn is equated to some quantity of iron, for instance 1 quarter of corn = x cwt of iron. What does this equation signify? It signifies that a common element of identical magnitude exists in two different things, in 1 quarter of corn and similarly in x cwt of iron. Both are therefore equal to a third thing, which in itself is neither the one nor the other. Each of them, so far as it is exchange-value, must therefore be reducible to this third thing.

3. A simple geometrical example will illustrate this. In order to determine and compare the areas of all rectilinear figures we split them up into triangles. Then the triangle itself is reduced to an expression totally different from its visible shape: half the product of the base and the altitude. In the same way the exchange-values of commodities must be reduced to a common element, of which they represent a greater or a lesser quantity.

4. This common element cannot be a geometrical, physical, chemical or other natural property of commodities. Such properties come into consideration only to the extent that they make the commodities useful, i.e. turn them into use-values. But clearly, the exchange relation of
commodities is characterized precisely by its abstraction from their use-values. Within the exchange relation, one use-value is worth just as much as another, provided only that it is present in the appropriate quantity. Or, as old Barbon says: 'One sort of wares are as good as another, if the value be equal. There is no difference or distinction in things of equal value... One hundred pounds worth of lead or iron, is of as great a value as one hundred pounds worth of silver and gold.'

(5) As use-values, commodities differ above all in quality, while as exchange-values they can only differ in quantity, and therefore do not contain an atom of use-value.

(6) If then we disregard the use-value of commodities, only one property remains, that of being products of labor. But even the product of labor has already been transformed in our hands. If we make abstraction from its use-value, we abstract also from the material constituents and forms which make it a use-value. It is no longer a table, a house, a piece of yarn or any other useful thing. All its sensuous characteristics are extinguished. Nor is it any longer the product of the labor of the joiner, the mason or the spinner, or of any other particular kind of productive labor. With the disappearance of the useful character of the products of labor, the useful character of the kinds of labor embodied in them will also disappears; this in turn entails the disappearance of the different concrete forms of labor. They can no longer be distinguished, but are all together reduced to the same kind of labor, human labor in the abstract.

(7) Let us now look at the residue of the products of labor. There is nothing left of them in each case but the same phantom-like objectivity; they are merely congealed quantities of homogeneous human labor, i.e. of human-power expended without regard to the form of its expenditure. All these things now tell us is that human-labor power has been expended to produce them, human labor has been accumulated in them. As crystals of this social substance, which is common to them all, they are values--commodity values. (Marx 1977, pp. 127-28)

Bohm-Bawerk's critique was centered mainly around paragraphs 2, 4, and 6. In reference to the second paragraph, Bohm-Bawerk argued that:
the first assumption, according to which an 'equality' must be manifested in the exchange of two things, appears to me to be very old fashioned, which would not, however, matter much were it not also very unrealistic. In plain words, it seems to me to be a wrong idea. Where equality and exact equilibrium obtain, no change is likely to occur to disturb the balance. (Bohm-Bawerk 1949, 68)

This argument against Marx's conceptualization of exchange relation as an equation has been repeated in different ways in almost all the subsequent criticisms of Marx.

Bohm-Bawerk's next arguments were directed against paragraphs 4 and 6, simultaneously. Marx, contended Bohm-Bawerk, having conceived of exchange as equality, then searches for a "common factor" in the following way:
He passes in review the various properties possessed by the objects made equal in exchange, and according to the method of exclusion separates all those which cannot stand the test, until at last only one property remains, that of being the product of labor. (69)
This method of derivation reminded Bohm-Bawerk of "one who urgently desiring to bring a white ball out of an urn takes care to secure this result by putting in white balls only." In other words, "he limits from the outset the field of his search for the substance of exchange-value to 'commodities,'...
and limits it to the products of labor against gifts of nature" (70).

However, according to Bohm-Bawerk, even if we accept this narrow definition of commodities, the logic of Marx is still false. In disregarding "the value in use of commodities," Marx confuses "abstraction from the genus, and abstraction from the specific forms in which the genus manifest itself."
The special forms under which the value in use of the commodities may appear, whether they serve for food, shelter, clothing, etc., is of course disregarded, but the value in use of the commodity as such is never disregarded. (74)
But this is not all, argued Bohm-Bawerk. "The second step" in Marx's argument "is still worse." "If the use-value of commodities be disregarded," Bohm-Bawerk quoted Marx, "there remains in them only one other property, that of being products of labor." But is this so, asked Bohm-Bawerk.
Is not the property of being scarce in proportion to demand also common to all exchangeable goods? Or that they are the subjects of demand and supply? Or that they are appropriated? Or that they are natural products? (75)

This argument too appears in almost all the subsequent criticisms of Marx's derivation.

All of these "obvious faults of logic and method" were attributed by Bohm-Bawerk to the artificial nature of the beginning of Capital (66). Marx was convinced that "labor is the source of value," said Bohm-Bawerk, thus leading him to concoct an argument, a "dialectical hocus-pocus," and to present it as though it was the result of a long period of thinking and investigation (77).

II. The history of Marx's derivation of value

Bohm-Bawerk was not the first critic of Marx's derivation of value. Even during Marx's own lifetime, some writers had expressed their dissatisfaction with his derivation, and Marx was well aware of these criticisms. This of course raises an important question: Why didn't Marx start Capital with "value," rather than beginning with "the commodity," and then trying to derive "value"? A meaningful answer to this question would require a thorough analysis of the development of Marx's concepts not only of "commodity" and "value," but of science and scientific method as well, particularly as this concerns the proper point of departure for political economy. Such an analysis, although helpful in dispelling the charge that the beginning of Capital was merely an artificial afterthought, lies beyond the scope of this paper. Suffice it to say that the issue of the point of departure is both posed and settled by Marx in his notebooks of 1857-58, better known now as the Grundrisse. Here, at the end of these writings, we encounter Marx's first attempt to formally derive the concept of value. In a section which still bears the title of "Value," it is stated that:

The first category in which bourgeois wealth presents itself is that of the commodity. The commodity itself appears as the unity of two aspects. It is use-value, i.e. object of the satisfaction of any system whatever of human needs. This is the material side, which the most disparate epochs of production have in common, and whose examination therefore lies beyond political economy. Use-value falls within the realm of political economy as soon as it becomes modified by the modern relations of production... Now how does use-value become transformed into commodity? Vehicle of exchange-value. (Marx 1973, 881)

So far, this attempt at the derivation of value is very similar to that which we saw earlier in Capital. At this point, however, the two texts begin to differ considerably, particularly when, within the discussion of the process of exchange, a digression appears on the historical development of exchange. Shortly thereafter, Marx's manuscript comes to an abrupt end, without either an
explanation of the meaning of "exchange-value" or any actual derivation of the concept of "value."
The second attempt at the derivation of value appears in A Contribution to the Critique of Political Economy, published in 1859. This derivation is very similar to the first attempt. There are, however, certain modifications. The title of the first chapter now reads "The Commodity," and the opening sentences are modified:
The wealth of bourgeois society, at first sight, presents itself as an immense accumulation of commodities, its unit being a single commodity. Every commodity, however, has a twofold aspect--use-value and exchange-value.

To begin with, a commodity, in the language of the English economists, is 'anything necessary, useful or pleasant in life', an object of human wants, a means of existence in the widest sense of the term. Use-value as an aspect of the commodity coincides with the physical palpable existence of the commodity. (Marx 1970, 27)

These opening sentences are followed by further discussion of the concept of "use-value" and, similar to the first attempt, the place of this concept in political economy. The analysis then leads to the relation between the concepts of "use-value" and "exchange-value," and the definition of "exchange-value":

Use-value is the immediate physical entity in which a definite economic relation--exchange-value--is expressed.

Exchange-value seems at first to be a quantitative relation, the proportion in which use-values are exchanged for one another. In this relation they constitute equal exchangeable magnitudes. (28)

Two sets of "use-values" are then used by Marx to elucidate the point. The first set is "snuff and elegies" and the second, "tins of boot polish" and a "palace." These peculiar sets are chosen to show that quite irrespective, therefore, of their natural form of existence, and without regard to the specific character of the needs they satisfy as use-values, commodities in definite quantities are congruent, they take one another's place in the exchange process, are regarded as equivalents, and despite their motley appearance have a common denominator. (28)

And what is this common denominator? The answer is given, rather imperceptibly, in the next two paragraphs:

Use-values serve directly as means of existence. But, on the other hand, these means of existence are themselves the products of social activity, the result of expended human energy, materialized labor. As objectification of social labor, all commodities are crystallization of the same substance. The specific character of this substance, i.e., of labor which is embodied in exchange-value, has now to be examined.

Let us suppose that one ounce of gold, one ton of iron, one quarter of wheat and twenty yards of silk are exchange-values of equal magnitude. As exchange-values in which the qualitative difference between their use-value is eliminated, they represent equal amounts of the same kind of labor. The labor which is uniformly materialized in them must be uniform, homogeneous, simple labor... Labor which creates exchange-value is thus abstract general labor. (28-9)

Even though the term "value" is not used in the above passages, the derivation of value is nonetheless complete at this point, since materialized "abstract general labor" is another expression for Marx's concept of "value."
This constitutes Marx's second attempt to derive value. The derivation is short and to the point, occupying no more than two and a half pages. From here Marx goes on to analyze various aspects of the concept of labor, such as the distinction between "concrete" and "abstract labor," and the measurement of labor.

The third derivation of value is found in the first edition of *Capital*, published in 1867, that is, almost eight years after the publication of *A Contribution*. The first few paragraphs of this derivation are very similar to the previous work. But even here there are some minor changes that should be noted. The title of the chapter is of course fixed now as "The Commodity." In the opening paragraph, Marx paraphrases the first sentence of *A Contribution* and proceeds directly to an analysis of the commodity, without mentioning the twofold aspect of the commodity, "use-value and exchange-value." The commodity now is defined as an "external object, a thing which satisfies through its qualities human needs of one kind or another" (Marx 1976, 7). Next, the concept of "use-value" is presented:

It is the utility of a thing for human life that turns it into a *use-value*. By way of abbreviation let us term the useful thing itself (or commodity-body, as iron, wheat, diamond, etc.) *use-value, good, article.* (7)

After some continued discussion of the concept of "use-value," and without any further mention of the place of this concept in the sphere of investigation of political economy, Marx proceeds to the concept of "exchange-value," repeating verbatim what he had earlier stated about this concept in *A Contribution*. At this point, however, the two texts begin to differ perceptibly. Immediately after defining "exchange-value," Marx casts doubt on the concept of "valeur intrinsique" by arguing that the relation of exchange "constantly changes in accordance with time and place," and thus the existence of an intrinsic value appears to be a "contradictio in adjecto" (8). In order to "examine the matter more closely," he then adds three new paragraphs that do not appear in *A Contribution*. These new paragraphs differ from paragraphs 1-3, quoted earlier from the third edition of *Capital*, only in certain minor details, too small to mention. The following three paragraphs, however, are substantially different from those of the third edition:

The fact that the substance of the exchange-value is something utterly different from and independent of the physical-sensual existence of the commodity or its reality as a *use-value* is revealed immediately by its exchange relationship. For this is characterized precisely by the *abstraction* from the *use-value*. As far as the exchange-value is concerned, one commodity is, after all, quite as good as every other, provided it is present in the correct proportion. Hence, commodities are first of all simply to be considered as *values*, independent of their exchange-relationship or from the *form*, in which they *appear* as exchange-values.

Commodities as objects of use or goods are corporeally different things. Their reality as *values* forms, on the other hand, their *unity*. This unity does not arise out of nature but out of society. The common social substance which merely manifests itself differently in different use-values, is *labor*. (8-9)

After these paragraphs Marx, declares commodities to be nothing, as values, but "crystallized labor." The derivation of value in the first edition of *Capital* is completed at this point. From here Marx goes on to discuss the unit of measurement of labor and the dual nature of labor. In relation to this latter issue, Marx repeats what he had stated at the beginning of *A Contribution*, but not actually in *Capital*, that is: "Originally, the commodity appeared to us as a two-sided entity, use-value and exchange-value" (11).
Before discussing the next derivation let me briefly explain why, I believe, Marx extended the derivation in Capital beyond that in A Contribution. Between publishing A Contribution and the first edition of Capital, Marx wrote his notes on the history of economic theory, better known now as Theories of Surplus Value. Certain passages in these notes indicate that the changes in the derivation may have been simply a reaction to Samuel Bailey's A Critical Dissertation on the Nature, Measure and Causes of Value. In this work, Bailey argued that although in the act of exchange "one A is in our estimation equal to two B," this is not an expression of "positive but relative esteem" (Bailey 1967, 2-3). In other words, value denotes "nothing intrinsic, but merely the relation in which two objects stand to each other as exchangeable commodities" (4-5). In support of this argument, Bailey drew an analogy between exchange-value and distance. According to this analogy, "a thing cannot be valuable in itself without reference to another thing, any more than a thing can be distant in itself without reference to another thing" (5). Marx devoted a considerable number of pages in his Theories to refuting Bailey's argument. He argued that Bailey does not understand the difference between the concepts "object," "product," and "commodity," as is evident from Bailey's interchangeable use of these concepts. According to Marx, "if only two products existed, the products would never become commodities, and consequently the exchange-value of commodities would never evolve either" (Marx 1971, 144). Therefore the exchange of two "objects" or "products" does not turn them into commodities, and hence there can be no talk of commensurability. In order to derive the concept of value, Marx argued, one must assume that A and B are commodities and this requires the regularity of exchange.

With regard to Bailey's analogy, Marx contended that the distance between two objects has nothing to do with the relation of the type $1A = 2B$. Instead of this "insipid" geometric analogy, Marx gave a geometric analogy of his own in which he attempted to compare the reduction of "exchange-values" of commodities to a common property to the reduction of the areas of parallelograms to a "common element" (143-44).

All of these arguments seem to find their way into the first few pages of Capital. The doubt cast on the concept of intrinsic value, the assumption of the regularity of exchange in the first paragraph, the equation of exchange in the second paragraph, and the geometric analogy in the third paragraph appear all to be responses to Bailey's arguments.

The next derivation of value is found in both the French edition, published in installments between 1872-75, and the second edition of Capital, published in 1872. In the postface to the second edition, Marx himself alludes to the changes made in the derivation vis-a-vis the first edition: "In Chapter 1, Section 1, the derivation of value by analysis of the equations in which every exchange-value is expressed has been carried out with greater scientific strictness" (Marx 1977, 94). He does not, however, elaborate on this issue. So what does he have in mind by "greater scientific strictness"?

A comparison of the second and the third editions of Capital reveals that the derivation is almost identical in the two works. The major difference is paragraph 1, which in the third edition is slightly longer than the corresponding paragraph in the two earlier editions. Thus, for all practical purposes the differences in the derivation of value between the first and the second editions are almost identical to the differences between the first and the third editions. Therefore, from what has been said earlier, we can conclude that the "greater scientific strictness" must refer to the changes that occur in the derivation after the paragraph containing the geometric analogy, since up to this point the two derivations are almost identical. After this, as we saw earlier, the derivation in the first edition is completed in three short paragraphs, while in the second and third editions this is done in
four, relatively more elaborate, paragraphs.

We have now examined Marx's derivation of value in its different stages and various shapes and can conclude that, contrary to Bohm-Bawerk's assertion, the derivation was not an artificial afterthought. What Bohm-Bawerk read in the third edition of Capital was at least twenty-five years in the making and the result of many years of thinking and rethinking. Bohm-Bawerk, who was completely unaware of this, simply assumed that the whole thing was a hoax and this assumption, like his other concerning the end of Capital, is factually incorrect.

However, the fact that the derivation of value in the third edition of Capital was the result of a prolonged effort does not necessarily make it right. Actually, it is in the process of looking back at this effort that some of the problems with the derivation become apparent. A comparative analysis of the texts mentioned above reveals that this derivation, particularly in the third edition of Capital, may not only be misleading but logically flawed. This is shown in the next section.

III. Semantics and logic

One of the greatest obstacles to understanding the logic of Marx's derivation of value is his semantics. The entire derivation seems to turn around such seemingly simple concepts as "commodity," "use-value," "exchange-value," "exchange relation," and "value." Yet these concepts, despite all that has been written on Marx's economic theory, remain to this day largely ambiguous.

In this section I would like to concentrate on three of these, showing how their ambiguities can adversely affect not only the reader's understanding of Marx's logic of the derivation but Marx's own logic as well. This discussion is then followed by an analysis of the paragraphs quoted earlier from the third edition of Capital.

A. Use-value

The evolution of the concept of "use-value" from Adam Smith to Marx is an interesting issue. However, since I have dealt with this subject elsewhere, I do not wish to go deeply into this issue here.vii By way of summary, however, I would like to say that Smith and, following him, David Ricardo considered "value in use" to be one meaning of the word "value"; the other meaning being "value in exchange."viii Moreover, both Smith and Ricardo used the expression "value in use" in the sense of usefulness or utility. Marx, up until the early part of the Grundrisse, considers "use-value" to be a "form of value," a concept similar to that of Smith and Ricardo (Marx 1973, 177). However, beginning from the middle of the Grundrisse Marx changes the meaning of the term. From here onward, use-value is no longer considered to be a form of value, but rather to be one aspect of the commodity, along with the other, exchange-value. Moreover, the term "use-value" is used, again from here onward, primarily in the sense of "a useful thing," rather than usefulness. This is perhaps best exemplified in "Notes on Adolf Wagner," where Marx states clearly that a commodity in its natural form is "a useful thing, alias a use-value" (Marx 1975, 198).ix But there are numerous other passages which show the term "use-value" being employed in the sense of a useful object, some of which have been already cited in this article. For example, as noted earlier, Marx writes in A Contribution: "Use-value as an aspect of the commodity coincides with the physical palpable existence of the commodity" (Marx 1970, 27). Or, in the first edition of Capital, he states: "By way of abbreviation let us term the useful thing itself (or commodity-body, as iron, wheat, diamond, etc.) use-value" (Marx 1976, 7).

Before going on any further, let me emphasize the distinction between the classical use of the term "use-value" and that of Marx. "Use-value" in the classical sense of "usefulness" expresses a relationship between an object and human needs. "Use-value" in Marx's sense of "a useful thing,"
however, refers to the object itself, but of course only insofar as the object, by virtue of its properties, satisfies some kind of human needs.

Marx, however, fails to state clearly and explicitly his shift in the meaning of the term "use-value." Moreover, the detection of such a shift is hampered by the following problems. First, in using the term in the sense of "a useful thing" sometimes the indefinite article "a" in front of "a use-value" is missing in Marx's writings, blurring the distinction between his concept of "use-value" and that of the classical economists. This is clearly evident in some of the passages quoted earlier. Second, Marx uses certain expressions in which use-value appears to mean usefulness rather than a useful object, even though it is the other way around. Take for example the expression "the use-values of commodities," which is found more than once in the derivation of value (e.g., Marx 1977, 126 and 128). One has a natural tendency to interpret this expression as meaning the usefulness of commodities, particularly if one reads Marx after reading the classical economists. Yet this is not what Marx intends. The "use-values of commodities" is rather his shorthand expression for the "use-value aspect of commodities," i.e., looking at commodities from the perspective of being useful objects rather than "exchange-values." Third, on a few occasions Marx does seem to use the term use-value in the classical sense of usefulness.

The above problems can adversely affect the reader's understanding of Marx's logic of the derivation of value. For example, let us look back at paragraph 4 in the derivation of value in Capital. The following statement, which is, as I will show later, the most important link in Marx's overall logic, appears here: "the exchange relation of commodities is characterized precisely by its abstraction from their use-values." How is one to understand this sentence? Does the expression "their use-values" refer to the use-value aspect of commodities or to the usefulness of commodities? In other words, is the relation of exchange characterized by abstraction from the useful object or the usefulness of the object? Obviously, this makes a great deal of difference. In the first case we abstract not only from the relation between the object and human needs but from the object itself altogether. In the second case, however, only the relation between the human needs and the object is abstracted from. So which one is it? There is, again, a natural tendency to interpret the expression "their use-values" to mean the usefulness of commodities and, as we shall see in the last section of this paper, this is how it usually has been understood. In other words, the usual interpretation of this sentence has been that Marx abstracts from the usefulness of commodities. But this interpretation is not correct. In the discussion of "exchange-value," I will show in greater detail that it is actually the useful object itself that Marx is trying to abstract from. For the time being, however, let me remind the reader that in the 1867 edition of Capital, the same sentence reads this way: "This [the exchange relation] is characterized precisely by the abstraction from the use-value." This sentence seems to support my interpretation, and if this is the case, the ambiguous expression "their use-values" in the later editions must refer not to the usefulness of commodities but to the use-value aspect of commodities.

It appears, therefore, that the sentence in the third edition of Capital should read as follows: The exchange relation of commodities is characterized precisely by its abstraction from the use-value aspect of the commodities, or, to put it simply, abstraction from useful objects themselves. Having resolved the ambiguity, however, the question now becomes this: What does Marx mean by "the exchange relation" and why does he say that this relation is characterized by abstraction from useful objects? The answer to this important question—which to the best of my knowledge has never been asked in the previous debates on the logic of Marx's derivation—should become clear after we have
analyzed Marx's concepts of "exchange relation" and "exchange-value."

Before proceeding to this analysis, however, I would like to call attention to an additional source of confusion in *A Contribution* and *Capital*. Following his derivation of value, Marx attempts to draw a parallel between this derivation and the process of arriving at the concept of "abstract labor." He thus writes, in *A Contribution*: "As exchange-values in which the qualitative difference between their use-values is eliminated, they [exchange-values of equal magnitude] represent equal amounts of the same kind of labor" (Marx 1970, 29). Similarly, he writes in *Capital* that "just as one is abstracting, in the case of the values of coat and linen, from the difference between their use-values, just so, in the case of the labor which these values represent" (Marx 1976, 14). The expression "their use-values" that appears in these statements has been discussed above and should no longer pose any problem: it simply refers to the use-value aspect of commodities. What is problematic in these statements is the word "difference." Earlier, Marx was talking about abstraction from useful objects. Now, however, he is talking about abstraction from the "difference" between useful objects. But these abstractions conceptually are not the same. In the first case, useful objects vanish altogether. In the second case, however, we are left, presumably, with the similarities of useful objects. So which abstraction does Marx have in mind? If we look at *Theories*, the answer seems to be only the first, and not the second, abstraction. Here, in response to Bailey, Marx argues that in the process of exchange one use-value is exchanged with "an infinite mass of other things which have nothing in common with it--and even if there are natural or other similarities between those things, they are not considered in the exchange" (Marx 1971, 128). This means, once again, that from Marx's perspective the exchange relation is characterized by total abstraction from useful objects and not simply abstraction from the difference between useful objects. Thus, it appears that the statements concerning the abstraction from the "difference between their use-values" are due either to Marx's carelessness or to his own conceptual confusion.

To summarize, Marx changes the meaning of the classical expression "use-value" from usefulness to a useful object; and when in the derivation of value he argues that the exchange relation of commodities is characterized by abstraction from "their use-values," what he has in mind is that the relation of exchange is characterized by abstraction from useful objects themselves. The reader, however, may misunderstand the above statement to mean that the exchange relation is characterized by abstraction from the usefulness of commodities. This misunderstanding is partly due to Marx's own failure to state clearly and explicitly his shift in the meaning of the term "use-value."

**B. The exchange relation**

The most striking and visible expression of the exchange relation in Marx is of course the so-called equation of exchange which we saw in paragraph 2 of *Capital*: "1 quarter of corn = x cwt of iron," translated by Marx to mean: "a given quantity of corn is equated to some quantity of iron." This conceptualization of the relation of exchange in terms of an equation, with the symbol "=" used in the sense of equality, has been criticized, one way or another, in almost all critiques of Marx's derivation. However, what seems to have escaped the critics is that this is not the only mode of representation of the exchange-relation that appears in Marx. The relation of exchange is also conceptualized by Marx in terms of the expression "is worth," and he has used the same symbol "=" to express this relation. This is evident in *Capital*, where the same type of equation "x commodity A=y commodity B" is translated by Marx to mean "x commodity A is worth y commodity B" (Marx 1977, 139). In addition to this concept, Marx uses, among others, the concepts of "as good
as," "replaceable," "congruence," "parity," and "identity" to express the relation of exchange.\textsuperscript{xv} These many concepts of the exchange relation and multiple use of the same symbol can be quite problematic if "\(\equiv\)" is employed in any strict mathematical sense. The symbol "\(\equiv\)" is used in mathematics to express a relation defined on the set of numbers, and if used in any other way, the two sides of the symbol must have the same dimension. This relation is an equivalence relation in the sense that it is reflexive, symmetric, and transitive. But consider the relation "is worth," which, as an expression that is actually used in practice, seems to be Marx's most reasonable expression of the exchange relation. Realistically, the two sides of this relation do not have the same dimension. Hence, this relation is not an equivalence relation since it is not reflexive. It therefore follows that the two relations "is equal" and "is worth," strictly speaking, are not identical and cannot be represented by the same symbol. The question then becomes: How strictly was Marx using the relation of equality? To answer this question we need to examine Marx's equation of exchange in greater detail.

It has been rightly argued in recent years that Marx's equation of exchange is a nonsensical equation: it makes no sense to say a given quantity of a use-value is equal to some quantity of another use-value.\textsuperscript{xvi} I would argue, however, that Marx himself seems to have been aware of this and his logic of the derivation appears to be based on this issue. In Theories he contends, again in reference to Bailey, that "if y yards of linen equal x lbs. of straw, this [implies] a parity between two unequal things--linen and straw--making them equal magnitudes" and that it "is not as straw and linen that they are equated, but as equivalents" (Marx 1971, 139).\textsuperscript{xvii} These statements show that Marx was aware of the fact that use-values cannot be physically equal to each other. But the statements also seem to show a great deal of confusion. He first writes an equation in which use-values are physically equated, then he argues that since these use-values are not physically equal, they must be equal as something else. But if use-values are not physically equal to each other, why write them as such?

The answer to the above question may lie partially in the writings of some of the classical political economists. These economists, when speaking of exchange, often stated that something is exchanged for, or can be exchanged for, something else.\textsuperscript{xviii} From the perspective of modern algebra, the relation of exchange embedded in such statements is simply the expression "is exchanged for" or "can be exchanged for," a relation which from now on I will symbolize by the letter E.\textsuperscript{xix} But this was not of course the way that classical economists viewed the exchange relation. The concepts of intrinsic value and exchange of equivalence led many of these economists to view the relation of exchange as a relation of equality. Even Bailey, who did not believe in the concept of intrinsic value, as we saw earlier, expressed the relation of exchange as such. Marx, I believe, inherited the classical concept of exchange-relation as a relation of equality and adopted it without any hesitation. However, I would like to argue in the next section that Marx's own concept of exchange-value, which is ill-defined, reinforces the idea that relation of exchange is one of equality.

C. Exchange-value

Smith and, again following him, Ricardo defined the "value in exchange" of a good as the "power of purchasing other goods."\textsuperscript{xx} This definition is of course quite ambiguous, since it is not clear what is meant by "power of purchasing." But the subsequent examples that they provided make it apparent that what they had in mind by "value in exchange" of a good was quantities of other goods that can be exchanged for it.
Marx also used the term "exchange value" at times in the same way that it was used by Smith and Ricardo. For example, consider the following statement in A Contribution: "The exchange-value of a palace can be expressed in a definite number of tins of boot polish" (Marx 1970, 28). Here, by exchange-value of a palace Marx clearly means certain quantities of tins of boot polish. This concept of exchange-value is in fact generalized in Theories, where in response to Bailey, Marx writes (in English) that "If we speak of the value in exchange of a thing, we mean in the first instance of course the relative quantities of all other commodities that can be exchanged for the first commodity" (Marx 1971, 128). Note that the relation of exchange, "can be exchanged for," or as I referred to it earlier, $E$, appears in this statement, as well.

But this is not the only sense in which the term "exchange-value" appears in Marx's writings. In the Grundrisse, as was mentioned earlier, Marx changed the meaning of the classical expression "value in use," and in so doing he also changed the meaning of the expression "value in exchange." Here, he began to view "use-value" as the "content" of a commodity and "exchange-value" as the "form" of this content (Marx 1973, 267). In this sense the "form," or "exchange-value," was a "relation" between use-values (267). This concept of exchange-value as a relation is also present in Marx's later writings. For example, consider this statement, which we saw earlier in A Contribution and Capital: "Exchange-value seems at first to be a quantitative relation, the proportion in which use-values are exchanged for one another." Here, it is apparent that by "exchange-value" Marx means a relation between use-values.

There is also an additional third sense in which Marx uses the term "exchange-value": a use-value that is being exchanged. This is most clearly expressed in the "Notes on Adolph Wagner," when Marx writes that insofar as a use-value is "a bearer of exchange-value...it is itself 'exchange-value'" (Marx 1975, 198).

In short, Marx uses the term "exchange-value" in three different senses: a quantitative relation between use-values, a use-value in the process of exchange, and the exchange-value of a use-value, i.e., quantities of other use-values that can exchange for the use-value in question. In the last two senses a use-value appears to both be and have "exchange-value."

But are these different concepts of exchange-value compatible with one another, and if not, how does Marx reduce one to another? Let me try to answer this question by first looking at the relation of exchange, $E$, embedded inadvertently in some of the classical, as well as Marx's own statements, and explore some feasible ways to redefine the concept of exchange-value. I will then return to Marx's different concepts of exchange-value.

Suppose three use-values $b$, $c$, and $d$, are or can be exchanged for one another in the following manner: $1b \, E \, 1c$ and $1b \, E \, 2d$. There are two distinct and mutually incompatible ways to define exchange-value. First, one can generally refer to any use-value that satisfies the relation of exchange, $E$, as an exchange-value. In this sense, use-values $b$, $c$, and $d$ are exchange-values. Second, one can define whatever comes after $E$ to be the specific exchange-value of whatever comes before $E$. In this sense, $1c$ is the exchange-value of $1b$ expressed in $c$ and $2d$ is the exchange-value of $1b$ expressed in $d$. A shorter way of writing these statements would be as follows:

1) $1c$ is the $c$ exchange-value of $1b$; and
2) $2d$ is the $d$ exchange-value of $1b$.

Note that in either definition, "exchange-value" expresses a relation, but it is not itself a relation. Furthermore, the two definitions of exchange-value are irreducible to one another, unless we resort to some careless generalizations to reduce the second definition to the first one. For example, if we
overlook the fact that exchange-values of 1b are expressed in different units, the above two statements would appear as such:

1') 1c is the exchange-value of 1b; and
2') 2d is the exchange-value of 1b.

If we also drop the notion of 1b altogether, we get:

1") 1c is exchange-value; and
2") 2d is exchange-value.

That is, 1c and 2d appear to be simply exchange-values, reducing my second definition of exchange-value to the first one. More importantly, these defective statements combined make it appear that the exchange-values, 1c and 2d are somehow identical.

Now, let us return to Marx. The relation of exchange is not clearly defined by him; as a result he refers to "exchange-value" as a relation and simultaneously as a use-value expressing a relation. Furthermore, his first definition of exchange-value already involves the kind of careless generalizations that I alluded to above; and, I believe, it is as such that he reconciles this definition with his third one. To see this, let us look back at this definition. We are told that by the "value in exchange of a thing, we mean ... the relative quantities of all other commodities that can be exchanged for the first commodity." This definition of Marx resembles my second definition of exchange-value, except that Marx has already left out the very fact that each exchange-value is expressed in a particular unit. According to this definition, by exchange-value of a use-value we can "mean" many things. For instance, using my example, by exchange-value of 1b we can mean both 1c and 2d.

Further generalization leads Marx to simply declare these relative quantities, i.e., 1c and 2d, to be exchange-values, leaving out any notion of the first use-value, whose exchange-value is expressed by these relative quantities, i.e., 1b. This reconciles the first and the third concepts of exchange-value and makes use-values both have and be exchange-values.

Lastly, since by exchange-value of a use-value we can "mean" different use-values and since these use-values are simply exchange-values, it follows that as exchange-values, these use-values are, to use some of Marx's expression, "identical," "as good as," "equal," etc., that is, 1c=2d. This strengthens the idea in Marx that the relation of exchange is one of equality.

In the next section, we will see all of the above problems appearing in the derivation of value in the third edition of Capital. But let me add here that the same problems can also be seen in the derivation of value in A Contribution, as well as in the first edition of Capital. For example, in A Contribution, immediately after defining exchange-value as a relation and discussing the exchange of some use-values, Marx writes: "Considered as exchange-value, one use-value is worth just as much as another, provided the two are available in the appropriate proportion" (Marx 1970, 28). Similarly, in the first edition of Capital he states: "As far as the exchange-value is concerned, one commodity is, after all, quite as good as every other, provided it is present in the correct proportion" (Marx 1976, 8-9).

In sum, Marx's concept of exchange-value is ill-defined. Indeed, there is more than one such concept in his writings and these concepts are not compatible with one another. Moreover, these fuzzy concepts reinforce the classical view of the exchange relation as a relation of equality.

Having now examined Marx's concepts of exchange relation and exchange-value, we can answer the question posed earlier in the section on use-value, a question which I believe to be the most important issue in Marx's derivation of value: why does Marx believe that the relation of exchange
is characterized by abstraction from use-values? The answer appears to be based on the following arguments. As exchange-values, Marx argues, use-values, in certain proportion, are equal to one another. Using my previous example, this means that as exchange-values, 1c=2d. However, as was pointed out in the previous section, Marx knows that use-values, i.e., useful objects, are not physically equal. That is, as use-values, 1c=2d. It therefore follows, Marx reasons, that the relation of exchange, "=", is characterized by abstraction from use-values!

D. The logic of the derivation in Capital

Let us now return to the controversial paragraphs in Capital, quoted and numbered earlier in this article. In view of what has been said, these first four paragraphs deserve particular attention and analysis, since all the problems with the definitions and logic come to the fore at this point, leading Marx to the convoluted idea that the relation of exchange is in reality characterized by abstraction from use-values. In the remaining three paragraphs, Marx uses this idea to justify his abstraction from use-values and to arrive at "value." Given, however, that a proper justification for such an abstraction is lacking, these remaining paragraph are, from a logical perspective, inconsequential, and therefore, deserve less attention than the first four paragraphs.

In paragraph 1 Marx writes that "a quarter of wheat for example, is exchanged for x boot-polish, y silk or z gold." Note, once more, that the relation of exchange "is exchanged for" is embedded in this statement without Marx's awareness. From this statement Marx concludes that "x boot-polish, y silk or z gold, etc., each represent the exchange-value of one quarter of wheat." Here, once again, the fact that x boot-polish, y silk, etc. are different expressions of the exchange-value of a quarter of wheat is overlooked. That is, by exchange-value of a quarter of wheat we can "mean" x boot-polish, y silk, or z gold. In the next sentence, all traces of "a quarter of wheat" seem to vanish and x boot-polish, y silk, etc. are generalized as exchange-values: "x boot-polish, y silk, z gold etc., must, as exchange-values, be mutually replaceable or of identical magnitude." Since within Marx's vocabulary the expressions "mutually replaceable" or "identical magnitude" are synonymous with "equal," this last statement is equivalent to saying that as exchange-values, x boot-polish = y silk = z gold.

Paragraph 2, which contains the celebrated equation of exchange, now appears to be nothing more than an extension of the first paragraph. The equation "1 quarter of corn = x cwt of iron" is simply a generalization of the above series of equations. From this single equation Marx then concludes that the two things must be "equal to a third thing." What is missing here is the convoluted argument that we saw in the previous section. That is, as use-values, 1 quarter of corn and x cwt of iron cannot be equal, but as exchange-values they are equal. Therefore, there must be a "common element," a "third thing," having nothing to do with wheat and iron as physical goods, which makes these exchange-values equal to one another. How does one arrive at this "third thing"? Marx's answer obviously must be, by setting aside wheat and iron as use-values.

Before giving his answer, however, in paragraph 3 Marx tries to argue by means of his geometric analogy that the "common element," which the "exchange-values of commodities" are reduced to, cannot be another use-value, as Bailey had argued, but must be something else. Note that in this paragraph Marx switches back to the concept of commodities, or use-values, having exchange-values rather than being exchange-values. As it stands, the argument in this paragraph is too short and unclear. But if this is read in conjunction with the original and the longer version of this analogy in Theories, the thrust of Marx's argument appears to be the following. The measurement of the areas of all rectilinear figures entails the splitting up of these figures into triangles. The area
of the triangle, however, is determined by half the product of its base, "b", and its altitude, "h". This "hxb/2", according to Theories, is the common "property" or the "common element" between the areas of rectilinear figures (Marx 1971, 144-45). Thus, Marx believes, expressing the areas of all rectilinear figures in the area of the triangle is of little use in geometry; one must go beyond this expression to locate the common element between the areas of such figures. In the same way, he likes to argue that the relative expression of the exchange-value of a use-value in another use-value is useless in political economy; one must go beyond this expression to locate what exchange-values have in common. This geometric analogy, which in both works is marred with all kinds of notational and conceptual errors, not only does not "illustrate" the issue of the "third element," but creates additional problems, for the following reasons. First of all, it is not always necessary to split up rectilinear figures into triangles in order to measure their areas; there are different ways of measuring the areas of rectilinear figures, such as integration. Secondly, it makes no sense to say that "hxb/2" is the common "property," or the "common element" between all rectilinear areas; "hxb/2" is merely a method of calculation of the area of a triangle. Thirdly, "hxb/2" ultimately expresses the area of a triangle in the area of a unit square, which by convention has a unit side and therefore a unit area. In other words, in axiomatic geometry all areas are, after all, expressed relatively. Yet this is precisely what Marx is trying to avoid in his argument about exchange-value, i.e., that the concept of exchange-value is something "purely relative."

Now comes paragraph 4, which contains the most crucial part of the argument, why we must set aside use-value to arrive at the common element. We are told at the very beginning of this paragraph that the "common element cannot be a geometrical, physical, chemical or other natural property of commodities." Given my analysis in the previous section, it is clear why this is so: As exchange-values, 1c=2d. However, as use-values, 1c=2d. Thus, as far as the exchange relation, "=", is concerned, we set aside use-values altogether. This means that we set aside all those natural properties which make c and d use-values; and, therefore, none of these properties can be the "common element." Marx, however, does not write all of this clearly. For example, he first writes: "Such properties come into consideration only to the extent that they make the commodities useful, i.e. turn them into use-values." This is then followed by the claim concerning the relation of exchange: "But clearly the exchange relation of commodities is characterized precisely by its abstraction from their use-values." Why this is so is explained only by: "Within the exchange relation, one use-value is worth just as much as another, provided only that it is present in the appropriate quantity." But this is only half of the argument, i.e., as exchange-values, 1c=2d. The other half, i.e., as use-values, 1c=2d, is not mentioned at all by Marx.

Note that without prior knowledge, it is virtually impossible to understand paragraph 4. For one thing, as was mentioned earlier, if we misinterpret the term "use-value" to mean usefulness--which, given Marx's lack of clarity, is a distinct possibility--no sense can be made out of any part of this argument. In fact, if "use-value" is not read in the sense of a useful object, it is impossible to understand why none of the natural properties of commodities can be the common element. Even if we pass this hurdle, it is not at all "clear" what Marx means by the "exchange relation" and why this relation is characterized by abstraction from use-value. Indeed, the fact that no previous writer has ever questioned Marx's claim concerning the relation of exchange can partly be explained by the abstruseness of this paragraph. Had this question been raised, the remaining paragraphs, which historically have been given greater attention, would lose much of their significance. As was mentioned earlier, it is inconsequential whether Marx's argument concerning the residue of such an
abstraction is sound or not, once we realize that there is no justification for the abstraction to begin with. Nevertheless, for sake of completeness, let me say a few words about the remaining paragraphs.

Paragraph 5 reiterates the idea that use-values must be set aside to arrive at the common element. This is evident from the argument that as "exchange-values" commodities "do not contain an atom of use-value."

Paragraphs 6 and 7, at last, deal with the process of abstraction and the actual derivation of value. Let me first present what I believe to be the thrust of Marx's argument in these paragraphs, and then discuss the problems involved. One can reduce these two paragraphs to the following set of statements: (1) Products of labor are also use-values; (2) abstraction from use-values entails abstraction from "the useful character of the products of labor"; (3) given, however, that the useful character of different kinds of labor are "embodied" in the useful character of the products of labor, abstraction from the latter entails abstraction from the former; (4) the former abstraction, in turn, entails abstraction from the "different concrete forms of labor," leaving us with abstract "congealed" labor; and (5) thus abstraction from use-values results in "congealed" abstract labor, that is, "value."

There are three kinds of problems with paragraphs 6 and 7. First, Marx, as usual, does not present the above argument in a clear and cohesive manner. For example, when at the beginning of paragraph 6 he states, "If then we disregard the use-value of commodities, only one property remains, that of being products of labor," he does mean it! The next statement is actually an attempt to qualify the previous one: "But even the product of labor has already been transformed in our hands." In other words, it is not exactly the "products of labor" that are left over after the abstraction, but something else. That something else, as I indicated above, is supposed to be "embodied" abstract labor, or "value." This only becomes apparent after a careful reading of these passages. Second, the entire discussion here concerning labor and its relation to its product is premature, since such a relation is only dealt with afterward, in the second section of the first chapter, entitled "The Dual Character of the Labor Embodied in Commodities," and in the first section of the seventh chapter, entitled "The Labor Process" (Marx 1977, 131-38; 283-92). Third, there are some concepts in these paragraphs that are also troublesome and are in need of semantic clarification. One such concept is, of course, the notoriously ambiguous expression, "labor embodied," which also appears in these same paragraphs as "congealed" labor and "crystals" of labor. Such problems make difficult, if not impossible, any meaningful analysis of the arguments involved in these paragraphs, without a prior comprehensive analysis of Marx's concepts of labor and its relation to its product.

This concludes my analysis of Marx's derivation of value. Before continuing to the final section, let me summarize my arguments in this section. Marx's derivation of value, despite his many revisions, is flawed after all. Some terms used by Marx, such as "use-value," are not clearly defined and can, therefore, adversely affect the reader's understanding of the logic of the derivation. But more importantly, some terms, such as "exchange-value" and "exchange-relation," are simply ill-defined, resulting in Marx's own confusion and fallacious reasoning.

IV. Revisiting Bohm-Bawerk's critique

In light of what has been said in the previous section it should not be difficult to see why much of what has been written by Bohm-Bawerk, and many others, on Marx's derivation of value is either inconsequential or irrelevant. To the best of my knowledge none of these writers has ever asked what I have contended to be the most important question with regard to this derivation: what does
Marx mean by the relation of exchange and why does he think that this relation is characterized by abstraction from use-value? Why this question has not been asked is also not difficult to understand. Marx's semantics has never been analyzed seriously in the previous debates on the derivation of value. Indeed, both sides of this debate often have paid no attention to the meaning of the terms used by Marx, and in some cases, have simply misinterpreted them entirely.

Consider, for example, Marx's expression "use-value." How this term is used by Marx and how this usage affects our understanding of the logic of the derivation have, so far as I know, never been pointed out. Most readers, particularly those who are familiar with the writings of the classical economists, have had a propensity to read the term in the sense of usefulness, overlooking the second and most predominant sense in which it is used by Marx, i.e., useful object. This is even true of the Dictionary of Marxist Thought, where Marx's term "use-value" has been defined, not according to Marx, but actually according to Adam Smith, as "usefulness" (Bottomore 1983, 86 and 504).

Some, as I have pointed out elsewhere, have gone even so far as to confuse this term with the neoclassical concept of "utility." This was not the case for Bohm-Bawerk, but it is clear from his critique that by Marx's "abstraction from their use-values" he understood abstraction from the usefulness of the commodities. This understanding resulted in the argument that we saw in the first section of this paper, i.e., Marx confused "abstraction from the genus, and abstraction from the specific forms in which the genus manifests itself." That is, the "special forms under which the values in use of the commodities may appear, whether they serve for food, shelter, clothing, etc., is of course disregarded, but the value in use of the commodity as such is never disregarded." This argument is totally irrelevant, since it is not the way that commodities "serve" us or are useful to us that Marx is abstracting from, but the useful object itself.

Those writers who have criticized Bohm-Bawerk's critique have not fared much better. Hilferding, for example, never noticed any difficulty with Marx's term "use-value" or Bohm-Bawerk's understanding of it. He simply said: "A use-value is an individual relationship between a thing and a human being" (Hilferding 1949, 131). Given this concept of use-value, Hilferding went on to produce a litany of irrelevant arguments against Bohm-Bawerk. Or consider a more recent critic of Bohm-Bawerk such as Meek. In his book on the labor theory of value, instead of giving a direct reference to the term "use-value," Meek gave only a cross reference to "utility" (Meek 1975, 331). Thus without ever mentioning the difference between Marx's term "use-value" and the concept of "utility," or realizing Bohm-Bawerk's error on this issue, he went on to argue that Marx was not concerned with "use-value as such" but only with what Bohm-Bawerk himself called "the special forms under which the values in use of commodities may appear" (160-61). In support of this argument Meek then quoted Marx in A Contribution, that is, "Entirely apart from their natural forms and without regard to the specific kind of wants for which they serve as use-values, commodities in certain quantities equal each other, take each other's place in exchange, pass as equivalents, and in spite of their variegated appearance, represent the same entity" (161). Here, Meek did not realize that the abstraction involved in this quotation is abstraction from the useful object and not the abstraction from the usefulness of the object which Bohm-Bawerk had in mind.

The situation becomes far worse when we consider Marx's term "exchange-value." What Marx means by this term and how this fuzzy concept affects his logic, to the best of my knowledge, has never been addressed in the debates on the derivation of value or, for that matter, in any general discussion of Marx's economic categories. In dealing with this term, most writers have simply
quoted Marx's varied and confused concepts. Some, such as the contributor to the Dictionary of Marxist Thought, apparently unable to make sense out of Marx's own concept, have instead resorted to Adam Smith's definition of exchange-value as "the power to command other commodities in exchange" (Bottomore 1983, 86 and 504).

Given all this, it is unlikely that Bohm-Bawerk, who had a very limited knowledge of Marx's works and concepts, would have realized the problem with the term exchange-value in Capital. He actually used the expression "exchange-values" synonymously with the "prices of the commodities" (Bohm-Bawerk 1949, 66). With such an understanding, he obviously could not have noticed the actual logical error in the derivation or the source of this error. Among the many different ways that Marx conceptualized the relation of exchange, Bohm-Bawerk noticed only one: the equation of exchange with the symbol "=" used in the sense of equality. And all Bohm-Bawerk had to say about this equation, as we saw earlier, was that it is "old fashioned," "unrealistic," and in plain words, a "wrong idea." But why a wrong idea? Because, he said, "[w]here equality and exact equilibrium obtain, no change is likely to occur to disturb the balance" (68). This argument is obviously nonsense, since the mathematical concept of equality has nothing to do with the concepts of "equilibrium" and "balance."

But if Marx's equation of exchange was a wrong idea of the exchange relation, what was the right idea? Bohm-Bawerk did not "dwell" on the issue and instead proceeded with his "critical investigation of the logical" process by means of which Marx arrived at the "common factor" (69). The upshot of this "critical investigation" was, as indicated earlier, a two part-argument. First, Bohm-Bawerk proposed that Marx should not have abstracted from usefulness in general, but from the specific usefulness of the product. The reasoning behind this argument was that insofar as the exchange relation of commodities is concerned, the specific usefulness of commodities "is of course disregarded." This argument of Bohm-Bawerk, even if we put aside his confusion about Marx's term use-value, makes no sense. He had previously dismissed Marx's equation of exchange as the wrong idea, so it is not at all clear what relation of exchange he had in mind and why this relation was characterized by abstraction from the specific usefulness of the commodities. Marx's idea of abstraction from use-values had certain, however erroneous, justification. Bohm-Bawerk's argument, on the other hand, seems to be void of any justification whatsoever.

The second part of Bohm-Bawerk's argument was that if we abstract from "the use-value of commodities" we can find, besides Marx's common property of "being products of labor," many other common properties. Among these, according to Bohm-Bawerk, is the property of being "natural products." This argument shows, once again, how little Bohm-Bawerk understood what he was reading. First of all, as I mentioned in the previous section, a careful reading of Marx's texts indicates that the common property, after abstraction from "the use-value of commodities" is supposed to be "congealed" abstract labor and not "products of labor." Second, it is absurd to say that if we set aside "the use-value of commodities," we can still be left with such residues as "natural products." But why did Bohm-Bawerk say such a thing? Simply because he was misinterpreting the term use-value to mean usefulness rather than a useful object.

In conclusion, much of Bohm-Bawerk's critique of Marx's derivation of value, as well as the subsequent counter-critiques of Bohm-Bawerk's critique, is completely irrelevant. Neither side of this issue has paid close attention to the exact meaning of Marx's term "use-value," and therefore both sides often have misinterpreted the meaning of this term. None of the combatants have realized that Marx's term
"exchange-value" is not properly defined and is used in different, incompatible, ways. Neither side has probed deeply into Marx's concept of "the exchange relation," its source, and its flaws. And as a result, no one has questioned the meaning and the correctness of the statement which is at the heart of Marx's derivation: "But clearly, the exchange relation of commodities is characterized precisely by its abstraction from their use-values."

REFERENCES


Notes

ii. See, for example, "Notes on Adolf Wagner" (Marx 1975).

iii. Until otherwise stated all italics are in the original.

iv. From here on, simply referred to as A Contribution.

v. From now on, simply referred to as Theories.

vi. On this point see also Rubin 1972, 107-10.


ix. From here on, all italics are mine.

x. I have often found it helpful to use set theoretic notations to clarify these concepts and the distinction between them. For example, if we let \( O = \{ o \} \) represent the set of objects; \( N = \{ n \} \), the set of human needs at any given time; and \( S \), the relation of satisfaction between the human needs and the objects, then the set of use-values \( U = \{ u \} \) can be defined as:

\[
U = \{ u \} = \{ o : o S n \}
\]

The classical economists' term "use-value" corresponds roughly to the relation of satisfaction \( S \). Marx's term "use-value," on the other hand, corresponds roughly to \( u \), a member of the set \( U \).

This mode of presentation can be helpful in clarifying a statement which we saw earlier in the Grundrisse: "Use-value falls within the realm of political economy as soon as it becomes modified by the modern relations of production" (also see Marx 1970, 28, for similar statements). What Marx is trying to say is that the set of use-values, \( U \), is not the proper subject of investigation of political economy. It becomes so, however, once we have introduced, on this set, the relation of exchange.

xi. See, for example, the following statements which appear in the discussion of the process of exchange in Capital: "For the owner, his commodity possesses no direct use-value...It has use-value for others" (Marx 1977, 179).

xii. This is not a problem of translation. The original German versions read differently. See Marx 1983, 19, and Marx 1989, 69.

xiii. Here I am using the translation of the first edition of Capital, since the same sentence in the latest English edition has been incorrectly translated as: "Just as, in viewing the coat and the linen as values, we abstract from their different use-values..." (Marx 1977, 135). The expression "their different use-values" should be replaced by "difference between their use-values." See Marx 1983, 25, and Marx 1989,
XIV. Marx of course wants to argue that this abstraction would result in a "residue," i.e., "value" (Marx 1977, 128).


XVI. See, for example, Krause 1982, 28-29; Carling 1984-5, 409; or Carling 1986, 60.

XVII. By "equivalents," Marx here means "equal values" (Marx 1971, 127).

XVIII. For example, Ricardo wrote that water and air have little or no value in exchange because "nothing can be obtained in exchange for them," but gold, on the other hand, has a great deal of value in exchange because it "will exchange for a great quantity of other goods" (Ricardo 1976, 5).

XIX. An analysis of such a relation does not concern us here.


XXI. There is of course another concept of exchange-value in Marx, i.e. the "form of manifestation" of value (Marx 1977, 152). But this concept does not concern us since it obviously appears after the derivation of value.

XXII. For example, in Capital, instead of saying the area of the triangle is reduced to half the product of the base and the altitude, Marx writes, "The triangle itself is reduced to ... half the product of the base and the altitude." At first, this may appear to be either an error in translation or a simple slip. But it is neither. The same kind of error appears also in Theories. Here, after having assumed that a "triangle A" and a "parallelogram B" have the same area, Marx writes "A=hxb/2" instead of writing the area of A=hxb/2. This notational error is then followed by a convoluted logic: "As areas, the triangle and the parallelogram are here declared to be equal, to be equivalents, although as a triangle and a parallelogram they are different" (Marx 1971, 144-45). These are obviously the same kind of notational and conceptual errors that we saw earlier in the discussion of exchange-values.

XXIII. In an earlier English translation this last sentence has been translated more accurately: "Then one use-value is just as good as another, provided only it be present in sufficient quantity" (Marx 1974, 37). For the original German, see Marx 1989, 69.

XXIV. Even The new Palgrave, which has many entries dealing with Marxist economics, has overlooked this unique and important concept in Marx and has discussed only the classical concept of use-value and the neoclassical concept of utility.


XXVI. Paul Sweezy, for example, repeats all of Marx's mistakes by saying that "exchange-value appears to be a quantitative relation between things," and at the same time maintaining that a commodity both possesses exchange-value and is an exchange-value (Sweezy 1970, 27-28).

XXVII. In The new Palgrave the term "exchange-value," which is not only an important concept in Marxian economics but also in classical political economy, does not even have a direct entry.