

1. This question is about the code and transmission channel described in the lecture notes on page 4 (the codewords are 000 and 111). Assuming that the codewords 000 and 111 are equally likely to be transmitted, prove that the probability we will decode correctly is indeed 97.2%.
2. Let C be a binary code of length n . Suppose we have a symmetric channel with probability p of an error during transmission of a digit with $0 \leq p \leq 0.5$ (see page 3). Prove that Nearest Neighbor Decoding is indeed Maximum Likelihood Decoding.
3. Prove that there does not exist a binary one-error correcting code of length 6 with 9 codewords.
4. Let n be an integer with $n \geq 2$. Let C be the binary code of length n consisting of all words containing an even number of ones. So if $n = 3$ then $C = \{000, 110, 101, 011\}$.

Find $|C|$ and $d(C)$.

5. Consider the ternary code $\{000, 111, 222\}$. Suppose we have a channel and a constant p such that for all distinct $a, b \in \{0, 1, 2\}$, we have that the probability $P(a|b)$ that the symbol a is received if the symbol b is transmitted, is equal to p (e.g., if $p = 5\%$ and we transmit the symbol 0, then there is a 5% chance we receive the symbol 1, a 5% chance we receive the symbol 2 and a 90% chance we receive the symbol 0).

We use the following decoding algorithm when receiving a word $y_1y_2y_3$:

- (a) If $y_1y_2y_3$ contains three different symbols, we declare a decoding error.
- (b) If $y_1y_2y_3$ contains at most two different symbols, we decode as aaa where a is the symbol that shows up at least twice in $y_1y_2y_3$.

So we declare a decoding error if we receive 120 but decode as 111 if we receive 121.

We transmit the codeword 000 and use the decoding algorithm to decode the received word.

- (a) Calculate the probability that we declare a decoding error. Evaluate this if $p = 5\%$.
 - (b) Calculate the probability that we decode correctly as 000. Evaluate this if $p = 5\%$.
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