

1. Does there exist a binary $(15, 17, 5)$ -code?
2. Does there exist a binary one-error correcting code of length 8 containing 29 codewords?
3. Prove that there does not exist a binary one-error correcting code of length 6 with 9 codewords.
4. Let $t \in \mathbb{N}$. Put $n = 2t + 1$. Prove that the code

$$C = \{\underbrace{00 \dots 00}_{n \text{ times}}, \underbrace{11 \dots 11}_{n \text{ times}}\}$$

is a perfect t -error correcting code of length n .

5. Let n be an integer with $n \geq 2$. Let C be the binary code of length n consisting of all words containing an even number of ones. So if $n = 3$ then $C = \{000, 110, 101, 011\}$.

Find $|C|$ and $d(C)$.

6. Consider the ternary code $\{000, 111, 222\}$. Suppose we have a channel and a constant p such that for all distinct $a, b \in \{0, 1, 2\}$, we have that the probability $P(a|b)$ that the symbol a is received if the symbol b is transmitted, is equal to p (e.g., if $p = 5\%$ and we transmit the symbol 0, then there is a 5% chance we receive the symbol 1, a 5% chance we receive the symbol 2 and a 90% chance we receive the symbol 0).

We use the following decoding algorithm when receiving a word $y_1y_2y_3$:

- (a) If $y_1y_2y_3$ contains three different symbols, we declare a decoding error.
- (b) If $y_1y_2y_3$ contains at most two different symbols, we decode as aaa where a is the symbol that shows up at least twice in $y_1y_2y_3$.

So we declare a decoding error if we receive 120 but decode as 111 if we receive 121.

We transmit the codeword 000 and use the decoding algorithm to decode the received word.

- (a) Calculate the probability that we declare a decoding error. Evaluate this if $p = 5\%$.
- (b) Calculate the probability that we decode correctly as 000. Evaluate this if $p = 5\%$.