

MATH 291T**Homework 2**Due on Tuesday 02/17/09

1. Prove that the vectors 11001, 01110 and 10111 are linearly dependent over $\{0, 1\}$ but linearly independent over $\{0, 1, 2\}$.

2. Let C be the binary code with generator matrix $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$.

- (a) Write down all the codewords in C .
- (b) Find a generator matrix in standard form for C .
- (c) Find a parity check matrix for C .
- (d) Find all the parameters for C (so n , k and d).

3. Let C be the ternary code with parity check matrix $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \end{bmatrix}$. Find the parameters of C (so n , k and d) without writing down all the codewords in C .

4. Let $r \in \mathbb{N}$. Prove that a binary $[2^r - 1, 2^r - 1 - r, 3]$ -code is perfect.

5. If \mathbf{x} and \mathbf{y} are binary vectors of length n , then we put $\mathbf{x} * \mathbf{y} = (x_1y_1, x_2y_2, \dots, x_ny_n)$. So if $\mathbf{x} = 10111$ and $\mathbf{y} = 11001$ then $\mathbf{x} * \mathbf{y} = 10001$.

Prove that $w(\mathbf{x} + \mathbf{y}) = w(\mathbf{x}) + w(\mathbf{y}) - 2w(\mathbf{x} * \mathbf{y})$ for all binary vectors \mathbf{x}, \mathbf{y} of length n .

6. Let $n \geq 2$ and C the binary code consisting of all words of length n containing an even number of ones (see HW 1 #5).

- (a) Prove that C is linear over $\{0, 1\}$ (Ex. 5 might be useful).
 - (b) Find the parameters of C (so n , k and d).
 - (c) Find a parity check matrix for C .
-
-