

1. Does there exist a binary $(15, 17, 5)$ -code?
2. Does there exist a binary one-error correcting code of length 8 containing 29 codewords?
3. Let $t \in \mathbb{N}$. Put $n = 2t + 1$. Prove that the code

$$C = \{\underbrace{00 \dots 00}_{n \text{ times}}, \underbrace{11 \dots 11}_{n \text{ times}}\}$$

is a perfect t -error correcting code of length n .

4. Prove that the vectors 11001, 01110 and 10111 are linearly dependent over $\{0, 1\}$ but linearly independent over $\{0, 1, 2\}$.

5. Let C be the binary code with generator matrix $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$.

- (a) Write down all the codewords in C .
- (b) Find a generator matrix in standard form for C .
- (c) Find a parity check matrix for C .
- (d) Find all the parameters for C (so n , k and d).

6. If \mathbf{x} and \mathbf{y} are binary vectors of length n , then we put $\mathbf{x} * \mathbf{y} = (x_1y_1, x_2y_2, \dots, x_ny_n)$. So if $\mathbf{x} = 10111$ and $\mathbf{y} = 11001$ then $\mathbf{x} * \mathbf{y} = 10001$.

Prove that $w(\mathbf{x} + \mathbf{y}) = w(\mathbf{x}) + w(\mathbf{y}) - 2w(\mathbf{x} * \mathbf{y})$ for all binary vectors \mathbf{x}, \mathbf{y} of length n .

7. Let $n \geq 2$ and C the binary code consisting of all words of length n containing an even number of ones (see HW 1 #4).

- (a) Prove that C is linear over $\{0, 1\}$ (Ex. 6 might be useful).
- (b) Find the parameters of C (so n , k and d).
- (c) Find a parity check matrix for C .

8. Let C be the ternary code with parity check matrix $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \end{bmatrix}$. Find the parameters of C (so n , k and d) without writing down all the codewords in C .