

1. Let  $C$  be the binary code word generator matrix  $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ .

- Set up a standard array for  $C$ .
- Write down the coset leaders of each coset.
- Use your standard array to decode the following words :
  - 1111
  - 1110
  - 1001

2. Let  $C$  be the binary code with parity check matrix  $\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$ .

- Set up a syndrome table for  $C$ .
- Use your syndrome table to decode the word 11010.

3. Let  $C$  be a binary  $[n, k]$ -code. Fix  $1 \leq i \leq n$ . Let  $C_i$  be the set of all codewords in  $C$  whose  $i$ -th digit is zero. So

$$C_i = \{x_1 \dots x_n \in C : x_i = 0\}$$

- Prove that  $C_i$  is linear.
- Prove that  $C_i$  is a subgroup of  $C$  of index at most two.
- Deduce that exactly one of the following holds:
  - The  $i$ -th digit in every codeword in  $C$  is zero.
  - The  $i$ -th digit in exactly half of the codewords is zero.

4. Let  $C$  be the binary code of length 16 containing the words  $x_1x_2 \dots x_{15}x_{16}$  such that in the matrix

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_5 & x_6 & x_7 & x_8 \\ x_9 & x_{10} & x_{11} & x_{12} \\ x_{13} & x_{14} & x_{15} & x_{16} \end{bmatrix}$$

every row and every column contains an even number of ones.

- Prove that  $C$  is a linear code.
- Find the dimension and minimum distance of  $C$ .
- List all possibilities to decode the following words, using Nearest Neighbor Decoding:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$