

1. Let C be a binary $[n, k]$ -code. Fix $1 \leq i \leq n$. Let C_i be the set of all codewords in C whose i -th digit is zero. So

$$C_i = \{x_1 \dots x_n \in C : x_i = 0\}$$

- Prove that C_i is linear.
- Prove that C_i is a subgroup of C of index at most two.
- Deduce that exactly one of the following holds:
 - The i -th digit in every codeword in C is zero.
 - The i -th digit in exactly half of the codewords is zero.

2. Let C be a binary $[n, k, d]$ -code.

- Prove that the sum of the weights of all the codewords in C is at most $n2^{k-1}$.
- Prove that $d \leq \frac{n2^{k-1}}{2^k - 1}$.
- Suppose that $2d > n$. Prove the *Plotkin Bound* :

$$|C| \leq \frac{2d}{2d - n}$$

3. The codewords in the first order Reed-Muller code $R(1, 3)$ are of the form

$$x_1x_2 \dots x_7x_8 = [a_3 \ a_2 \ a_1 \ a_0] G(1, 3)$$

- Find the parity check sums for a_1 , a_2 and a_3 .
- Decode 10000011.
- Decode 10101010.

4. Consider the binary matrix

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Show that G is a generator matrix for the second order Reed-Muller code $R(2, 3)$.

5. Explain the probabilities on page 30 in the notes (I updated the notes. The correct percentages are 26.49%, 0.6929%, 0.0139% and 99.99%. First write your answers in terms of a general symbol-error-probability p . Then evaluate your answers (you will need a computer to find 99.99%).