

Entropy-Driven Exploration and Exploitation in Evolutionary Algorithms

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Outline of the Presentation

- Motivation
- Related Work
- Entropy in EA
- Results
- Conclusion

Motivation

To explore or to exploit?
This is the question now.

(anonymous EA agent)

Motivation

- Exploration is a process of visiting entirely new regions of a search space and of seeing if anything promising may be found in the regions.
- Exploitation is a process of using information gathered from the previously visited points in the search space to determine which regions might be profitable to be visited next.

Motivation

- How the balance between exploration and exploitation is achieved in EAs?
- How the balance between exploration and exploitation can be controlled?

Motivation

- In EAs, the selection process and operators (e.g., crossover and mutation) establish a balance between the exploration and exploitation of the search space.
- What about population size?

Motivation

- The selection process drives the searching towards the regions of the best individuals. Hence, selection can be mainly seen as an exploitation operation.
- However, selection can control the level of exploration or exploitation by varying selection pressure. Higher/lower selection pressure pushes the search towards more exploitations/exploration.

Motivation

- The mutation operator randomly modifies individuals, with a given probability, and thus increases the structural diversity of the population. From this point of view, the mutation operator is more an exploration operator.
- On the other hand, mutation can also be seen as an exploitation operator, because most of the genetic material is preserved.

Motivation

- The crossover operator combines two or more parents to generate better offspring. From this point of view, the crossover operator is more an exploitation operator.
- However, a good crossover operator should also generate individuals in the exploration zone.

Motivation

- The good balance between exploration and exploitation is achieved separately, but implicitly, in each operator (selection/mutation/crossover) or
- the balance should be coordinated among different operators (e.g., with high selection pressure such mutation/crossover should be chosen that is biased more to exploration).

Motivation

- In this work balance between exploration and exploitation is entropy-driven.
- The exploration/exploitation of the search space is adapted on-line based on the current status of the evolutionary process.

Related Work

- Optimal balance between exploration and exploitation has been, up to now, mainly controlled by determining the best control parameter values (tuning, deterministic, adaptive, self-adaptive).
- Empirical studies
 - De Jong (pop_size = 50, $p_m=0.001$, $p_c=0.6$)
 - Grefenstette (pop_size = 30, $p_m=0.01$, $p_c=0.95$)
 - Schaffer ($p_m \in [0.005, 0.01]$, $p_c \in [0.75, 0.95]$)
- Theoretical studies
 - Mühlenbein ($p_m=1/L$)
 - Hesser&Männer ($p_m(t)=\sqrt{\alpha/\beta} \cdot \exp(-\gamma \cdot t/2) / \text{pop_size} \cdot \sqrt{L}$)
 - Harik (pop_size = $-2^k \cdot \sqrt{\pi m \sigma^2} / 2d \cdot \ln \alpha$)

Related Work

- To maintain genetic diversity and hence providing better balance between exploration and exploitation various diversification techniques have been proposed: scaling mechanism, non-standard selections, incest prevention, neighborhoods, islands, ...

Related Work

- One of the earliest researchers that investigated entropy in EAs was Rosca (1995).
- His experiments showed that populations appeared to be stuck in local optima when entropy did not change or decrease monotonically in successive generations.

Related Work

- Ursem (2002) proposed the Diversity-Guided Evolutionary Algorithm (DGEA).
- DGEA uses a distance-to-average-point measure to alternate between phases of exploration and exploitation.
- However, DGEA does not use entropy as a measure for diversity.

Entropy in EAs

- Entropy is a concept in information theory, thermodynamics, and statistical mechanics.
- The basic concept of entropy in information theory has to do with how much randomness is in a signal or random event (X).
- Shannon: $H(X) = -\sum p_i \log_2 p_i$

Entropy in EAs

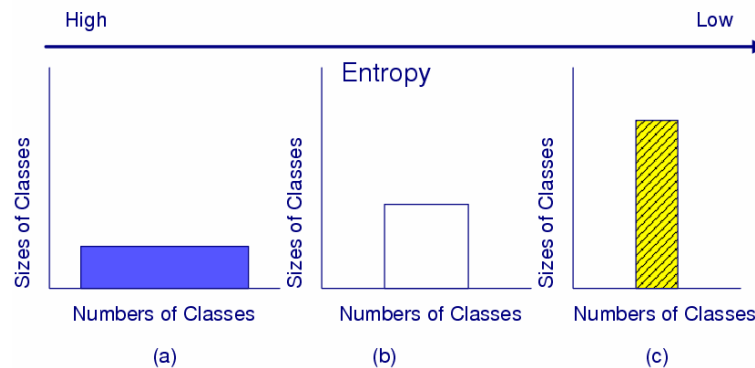
- Statistical mechanics explains entropy as the amount of uncertainty which remains in a system.
- Entropy is a function of the probability that the system is in various quantum states.
- The more states available to the system with higher probability, the greater the disorder and thus, the greater the entropy. If the system has only one possible state, there is no uncertainty, and the entropy of the system is zero.

Entropy in EAs

- As such, entropy is a succinct measure of diversity.
- Biological diversity refers to the differences between individuals in a population, which in nature imply structure (genotype) and behavior (phenotype) differences.

Entropy in EAs

- High entropy in EAs reveals the presence of many unique fitness values, where the population is evenly distributed over those values.



Entropy in EAs

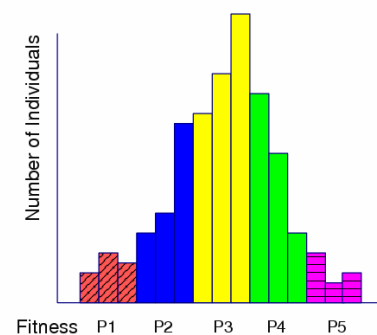
- Rosca calculates entropy for a population by first placing fitness values into fitness classes p_i and counting the size of each fitness class.
- The p_i is the proportion of the population occupied by the population partition i .
- Entropy = $-\sum p_i \log_2 p_i$

Entropy in EAs

- This work extends Rosca's work to experiment with entropy, using different flexible cases of fitness classes.
- Moreover, using entropy as a measure and programmable parameter control by PPC_{EA} (BIOMA'04), we are able to control exploration and exploitation in an adaptive manner.

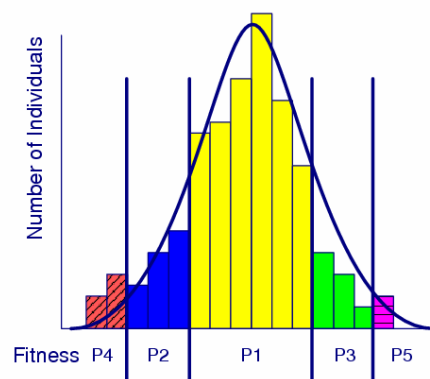
Entropy in EAs

- Linear: Assign a predefined yet changeable value to the number of fitness classes, n .
- For each generation, the interval between the best and worst fitness values is evenly partitioned into n sub-intervals as fitness classes.



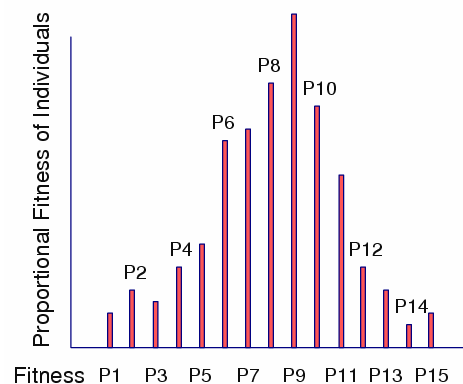
Entropy in EAs

- Gaussian: The partition of fitness classes in this case is derived from Gaussian distribution.
- For each generation, fitness classes are spread out from the average fitness value with the standard deviation.



Entropy in EAs

- Fitness proportional: Fitness proportional approach is a variation of Rosca' approach. Rosca's fitness classes are partitioned by individuals having same phenotypes.
- In our case p_i is the selection probability of i -th individual.



Results

Benchmark functions (Yao, 1999):

- High dimensional unimodal functions:

f1, f2, f4, f6, f7

- High dimensional multimodal functions with many local minima:

f9, f10, f11

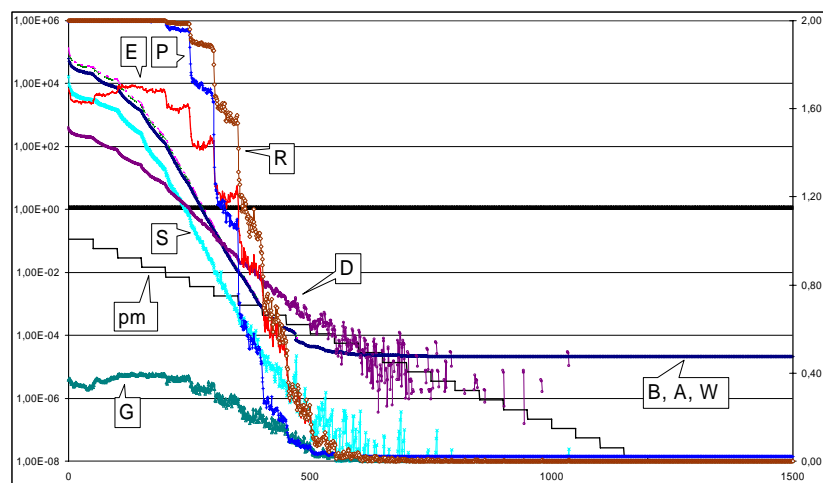
- Low dimensional multimodal functions with few local minima:

f14, f16, f17, f20

Results: f1

```
genetic
// Fogarty
Round := 50;
g := 0;
Epoch := 50;
Maxgen := 1500;
while ( g < Round ) do
  pm := 0.005;
  pc := 0.75;
  t := 0;
  temp := 1.0;
  init;
  while ( t < Maxgen ) do
    pm := ( 1 / 240 ) + ( 0.11375
    / temp );
    callGA;
    temp := temp * 2.0;

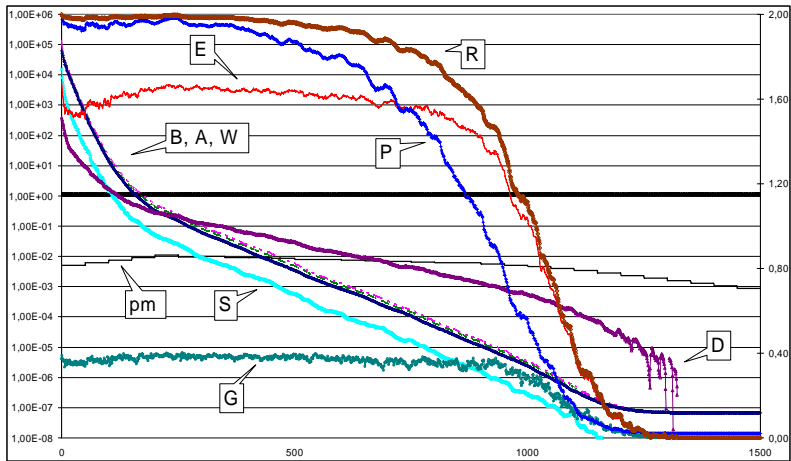
    t := t + Epoch
  end;
  g := g + 1
end
end genetic
```



Results: f1

```

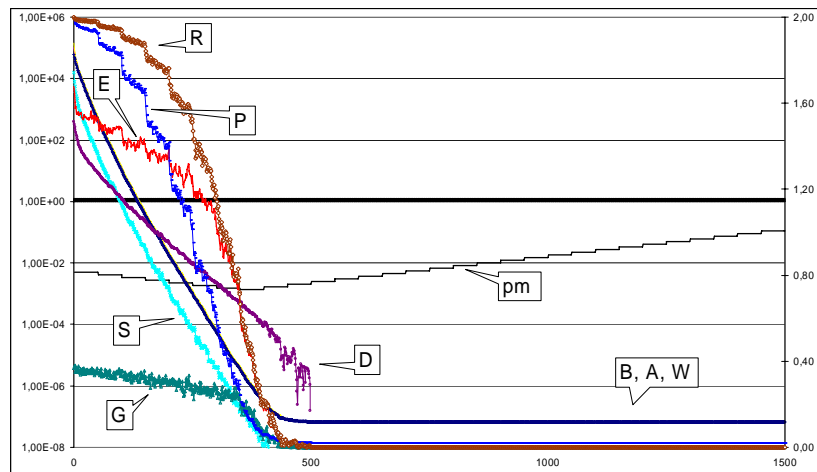
genetic // 1/5 success rule
Round := 50;
g := 0;
Epoch := 50;
Maxgen := 1500;
while ( g < Round ) do
  t := 0;
  pm := 0.005;
  pc := 0.75;
  init;
  while ( t < Maxgen ) do
    callGA;
    if ( RatioM > 0.2 ) then
      pm := pm * 1.22
    fi;
    if ( RatioM < 0.2 ) then
      pm := pm * 0.82
    fi;
    t := t + Epoch
  end;
  g := g + 1
end
end genetic
  
```



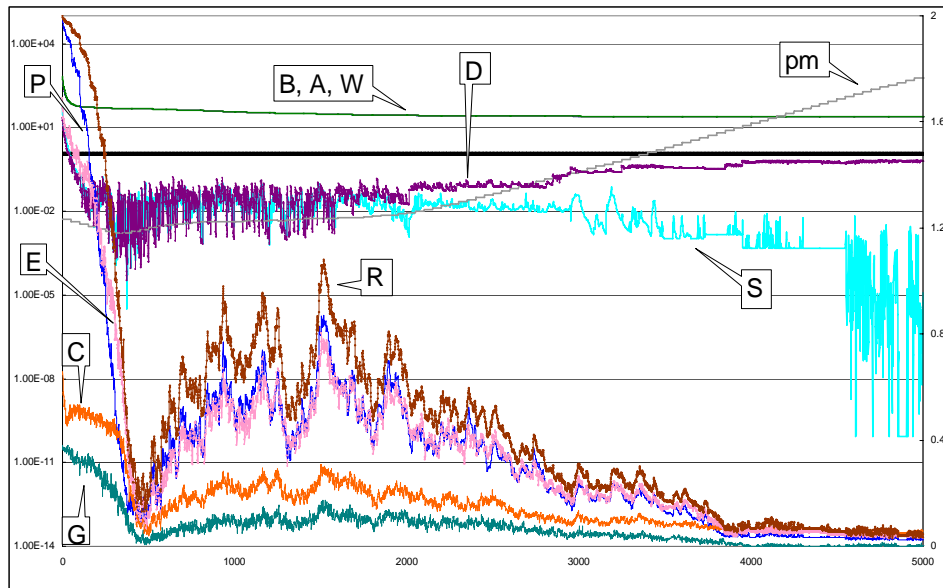
Results: f1

```

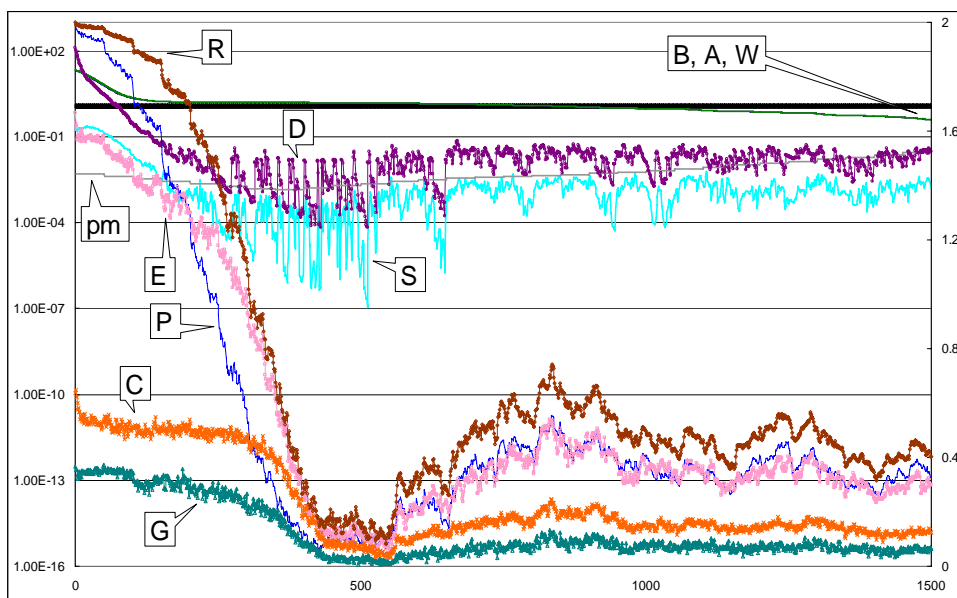
genetic
Round := 50;
g := 0;
Epoch := 50;
Maxgen := 1500;
while ( g < Round ) do
  pm := 0.005;
  pc := 0.75;
  t := 0;
  init;
  while ( t < Maxgen ) do
    callGA;
    if ( Entropy > 0.5 ) then
      pm := pm * 0.82
    fi;
    if ( Entropy < 0.5 ) then
      pm := pm * 1.22
    fi;
    t := t + Epoch
  end;
  g := g + 1
end
end genetic
  
```



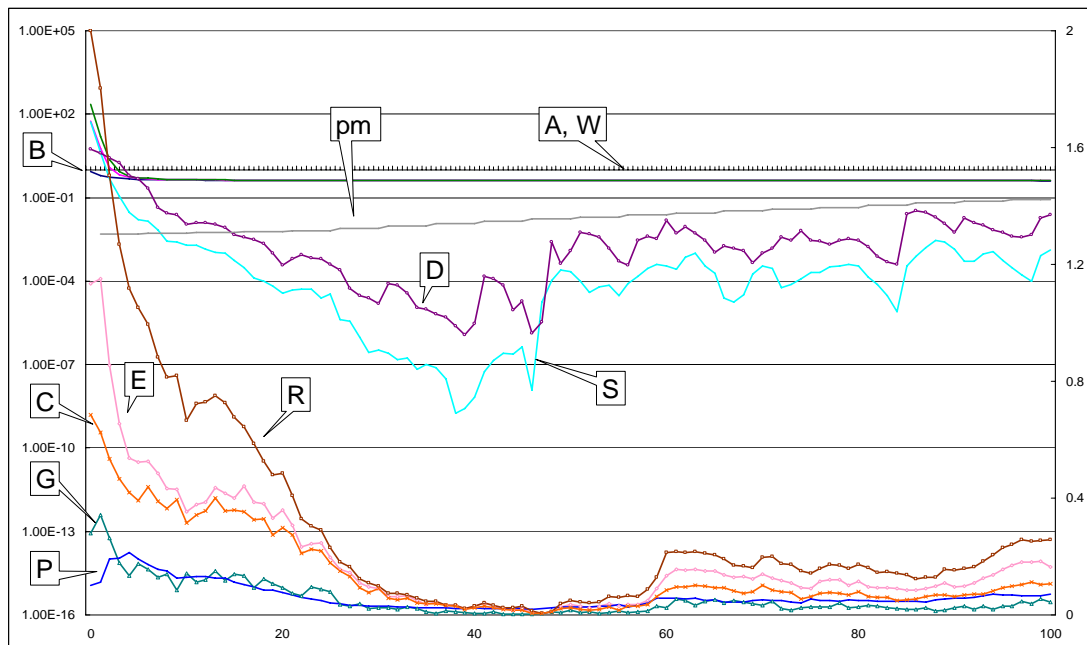
Results: f9 – Entropy-driven



Results: f10 – Entropy-driven



Results: f17 – Entropy-driven



Conclusion

- A novel entropy-driven exploration and exploitation approach has been presented.
- The on-line adaptation mechanism involves PPC_{EA} as to whether more exploitation or exploration is needed depending on the current progress of the algorithm and on the current estimated potential of discovering better solutions.