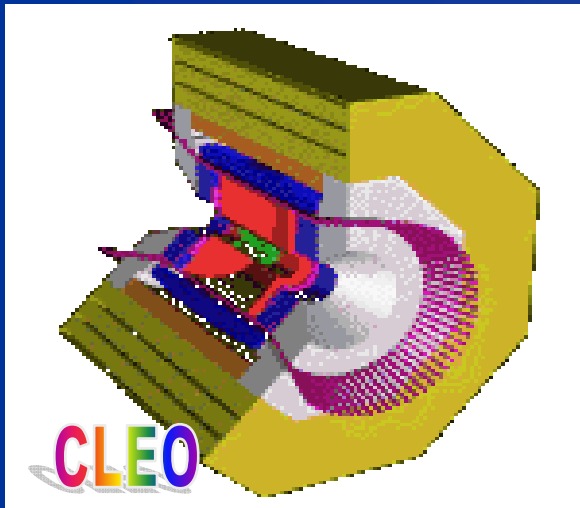


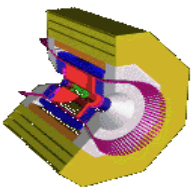
# Semileptonic Results from CLEO

Yongsheng Gao

Southern Methodist University  
(CLEO Collaboration)

ICHEP06, Moscow, July. 26 – Aug 2, 2006





# CLEO Semileptonic Results



**CLEO-c data: 281 pb<sup>-1</sup> at  $\psi(3770)$**

**D<sup>0</sup>/D<sup>+</sup> inclusive semileptonic decays**

**First Observation of  $D^+ \rightarrow \eta e^+ \nu$ ,  $D^0 \rightarrow K^- \pi^+ \pi^- e^+ \nu$**

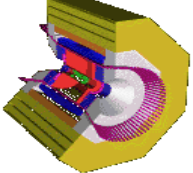
**Form Factors & Vcs, Vcd from  $D^0/D^+ \rightarrow K/\pi e^+ \nu$**

**First measurement of Form Factors in  $D \rightarrow \rho e^+ \nu$**

**Form Factors in  $D^+ \rightarrow K^- \pi^+ e^+ \nu$**

**Exclusive semileptonic  $b \rightarrow u$**

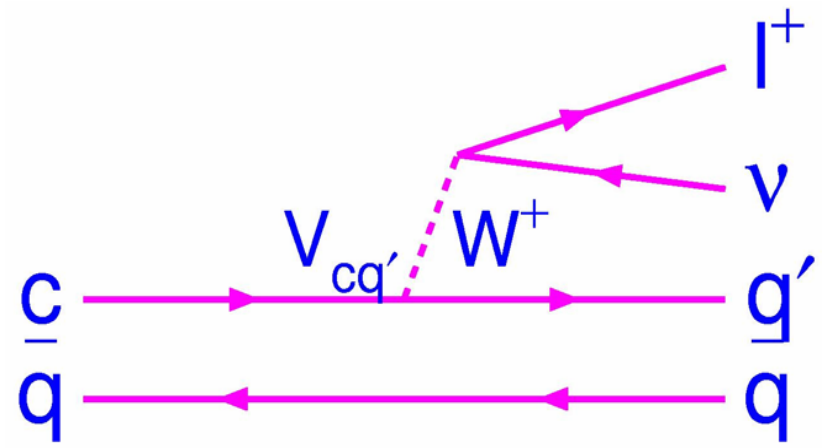
**CLEO Y(4S) data**



# Why Semileptonic D Decay?



$$\mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



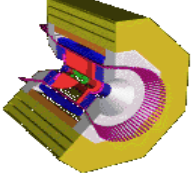
**$D^0 \rightarrow X e^+ \nu$  and  $D^+ \rightarrow X e^+ \nu$ :**

– **Inclusive semileptonic BR and spectrum**

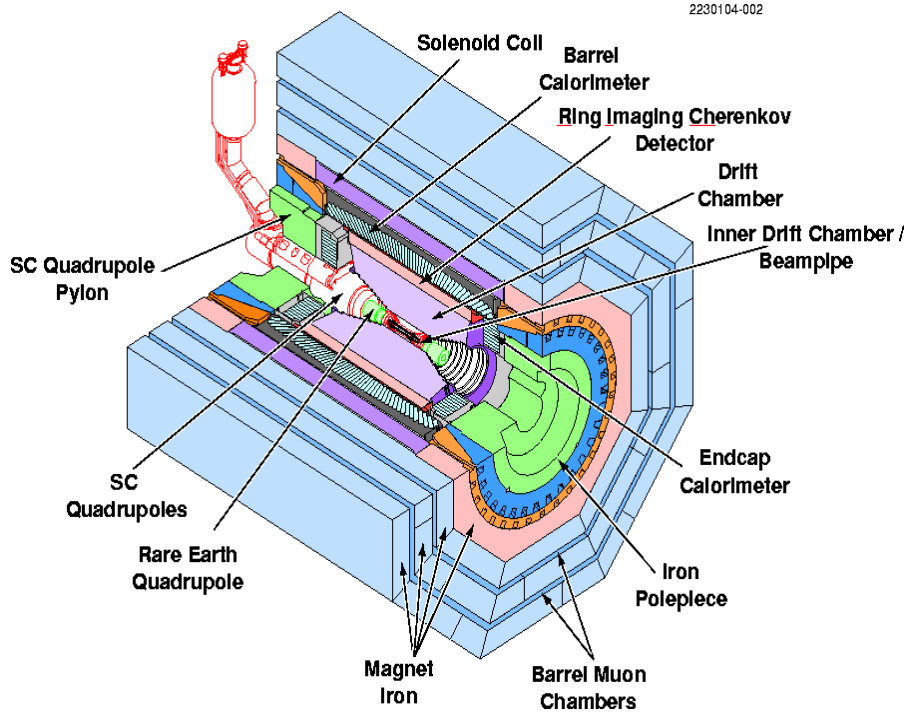
**$D^0 \rightarrow K^- e^+ \nu, \pi^- e^+ \nu$ , etc:**

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{cq'}|^2 p_P^3 |f_+(q^2)|^2$$

– **Form Factors,  $V_{cd}$ ,  $V_{cs}$  and  $V_{ub}$**



# CESR and CLEO-c

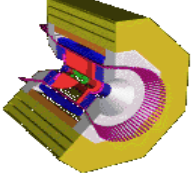


Tracking: **Drift Chambers**

Electron ID: **CsI Cal.**

Hadron ID: **RICH**



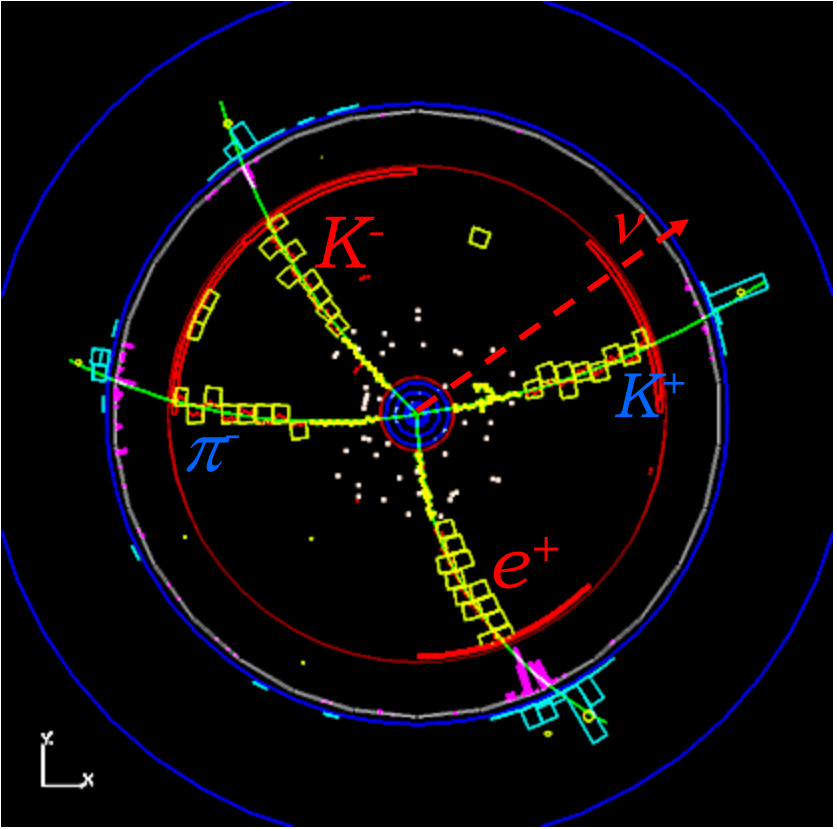
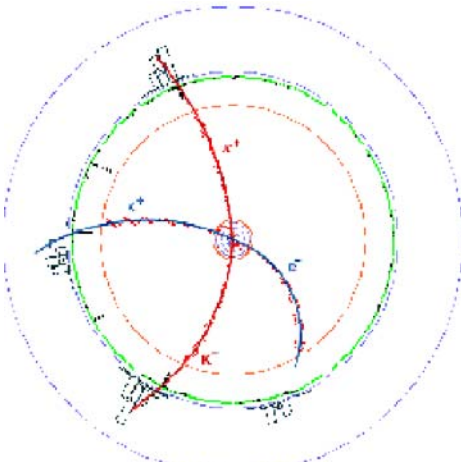
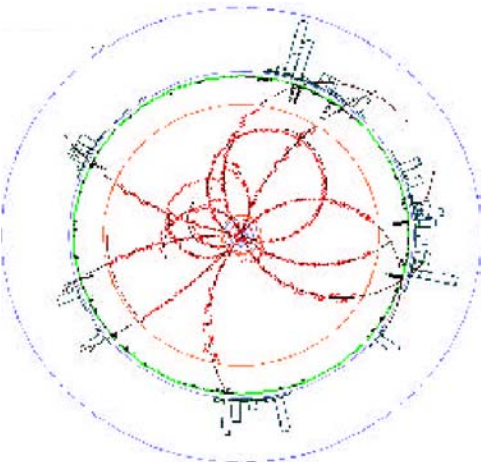


# Charm at $\Psi(3770)$ vs $\sim\Upsilon(4S)$



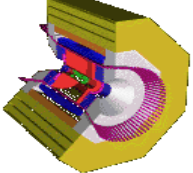
$\sim\Upsilon(4S)$

$\Psi(3770)$

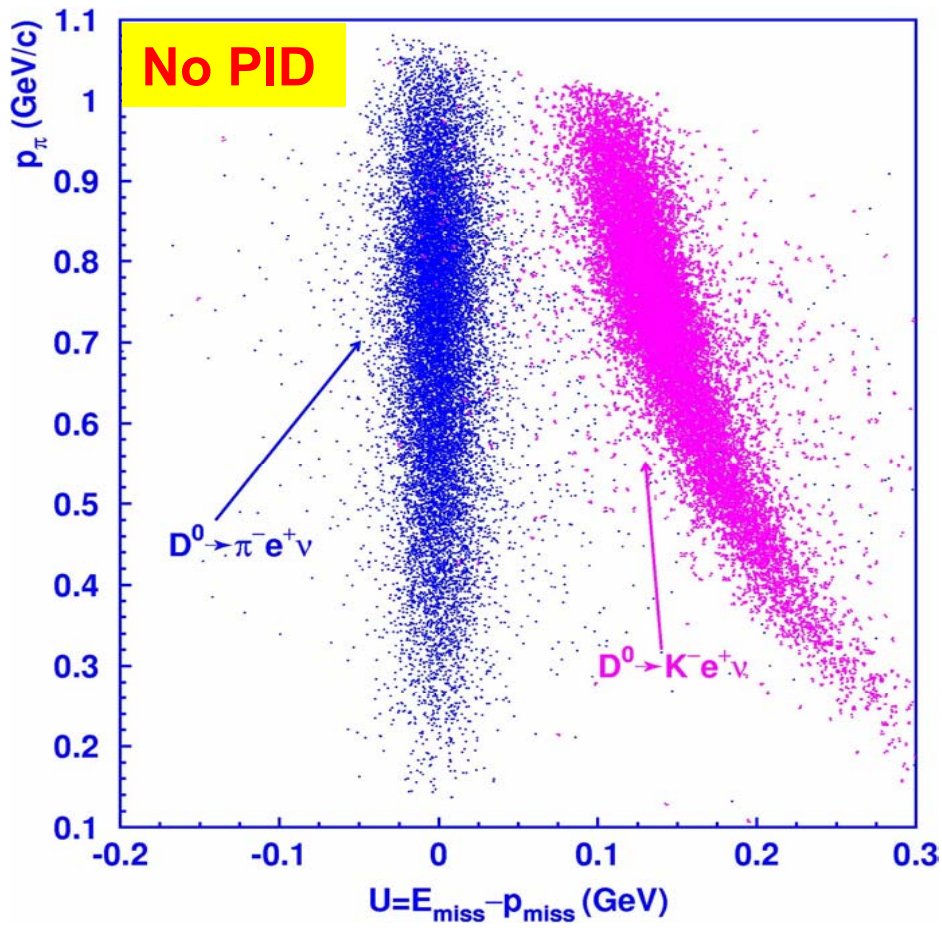
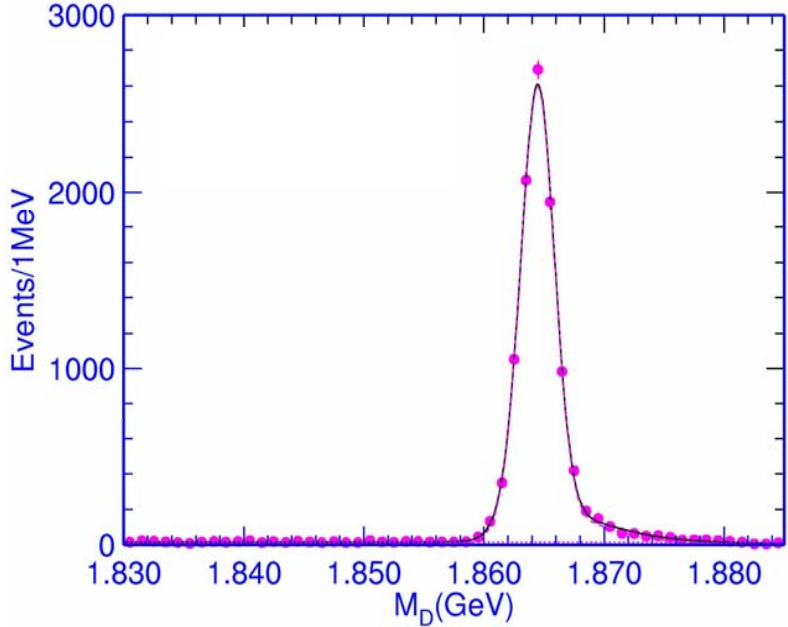
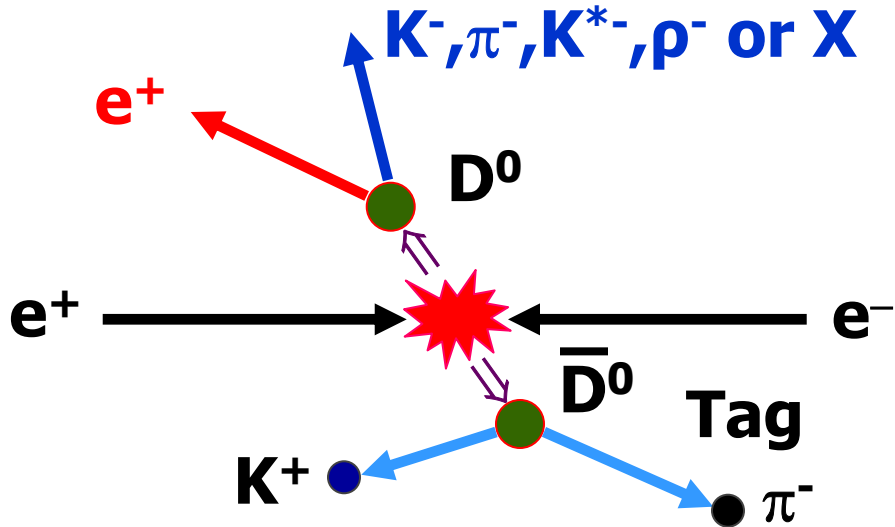


- Large Cross-Section
- Low Multiplicity
- NO Fragmentation
- Kinematics Variables:  $\rightarrow$
- "Background Free"

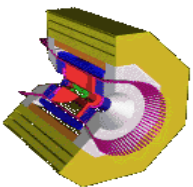
$$\left\{ \begin{array}{l}
 M_D \equiv \sqrt{E_b^2 - |p_D|^2} \\
 \Delta E = E_b - E_D \\
 U = E_{\text{miss}} - P_{\text{miss}}
 \end{array} \right.$$



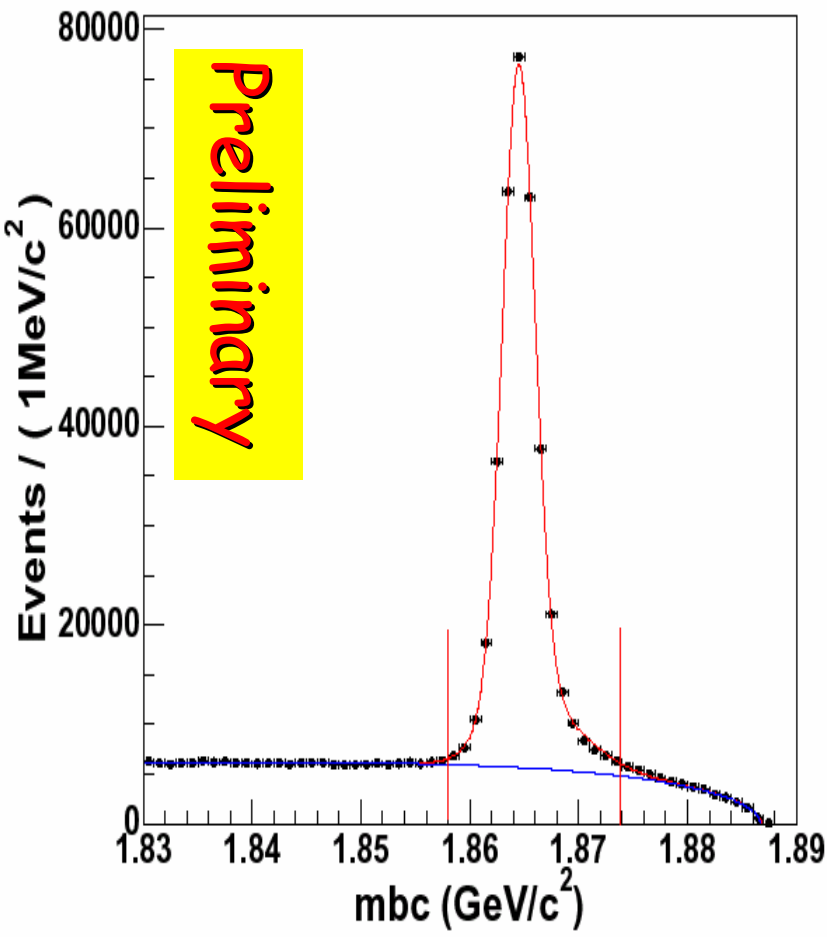
# Unique Kinematics at $\Psi(3770)$



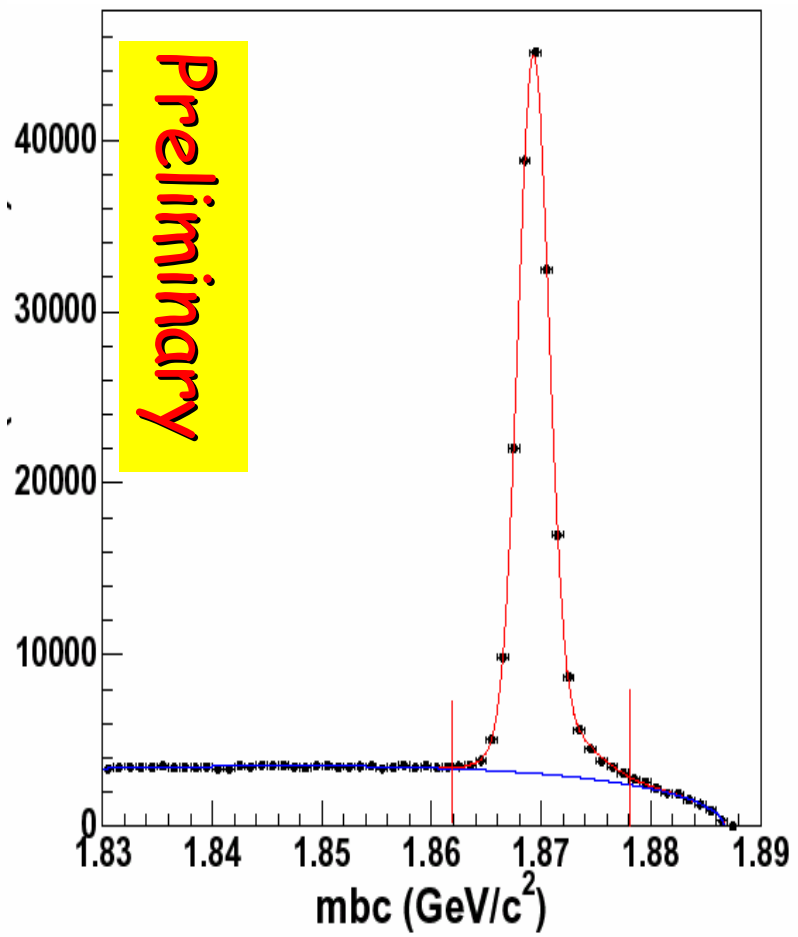
$D^0 \rightarrow \pi^- e^+ \nu$



# Fully Reconstructed $D^0/D^+$ (Tag)

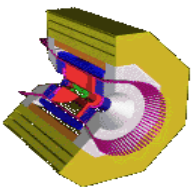


**All  $D^0$  tags**  
**~308K fully reconstructed**



**All  $D^+$  tags**  
**~163K fully reconstructed**

**From 281 pb<sup>-1</sup> at  $\Psi(3770)$**



# Inclusive Semileptonic Decays

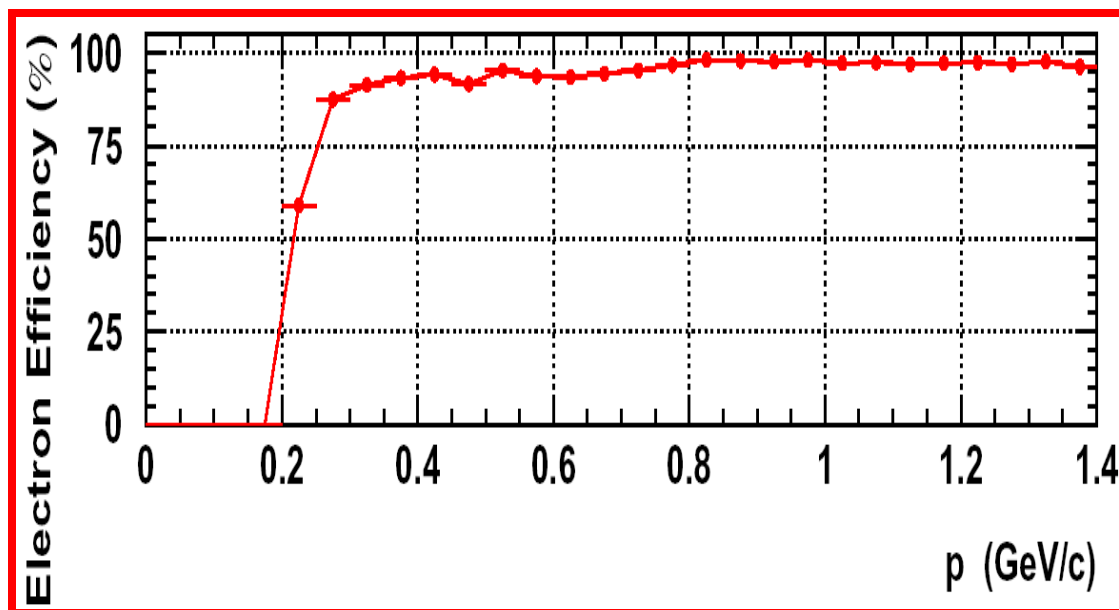


## Motivation

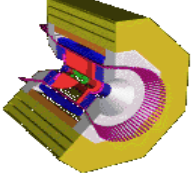
- BR( $D \rightarrow Xl\nu$ ) to compare with sum of exclusive BRs.
- Precision measurement of lepton momentum spectrum.
- Compare  $\Gamma_{sl}(D^0)/\Gamma_{sl}(D^+)$
- Test HQT with  $\Gamma_{sl}(D^0)/\Gamma_{sl}(D_s)$

## Technique

- D-Tag
- Electron ID
- Gold DTags only
  - $K^-\pi^+$  and  $K^-\pi^+\pi^+$
- Charge correlation







# Inclusive Semileptonic Results



mode	Branching Fraction
$D^0 \rightarrow Xe^+\nu$	$(6.46 \pm 0.17 \pm 0.13)\%$
$\Sigma_i B_i (D^0 \rightarrow Xe^+\nu)$	$(6.1 \pm 0.2 \pm 0.2)\%$
$D^+ \rightarrow Xe^+\nu$	$(16.13 \pm 0.20 \pm 0.33)\%$
$\Sigma_i B_i (D^+ \rightarrow Xe^+\nu)$	$(15.1 \pm 0.5 \pm 0.5)\%$

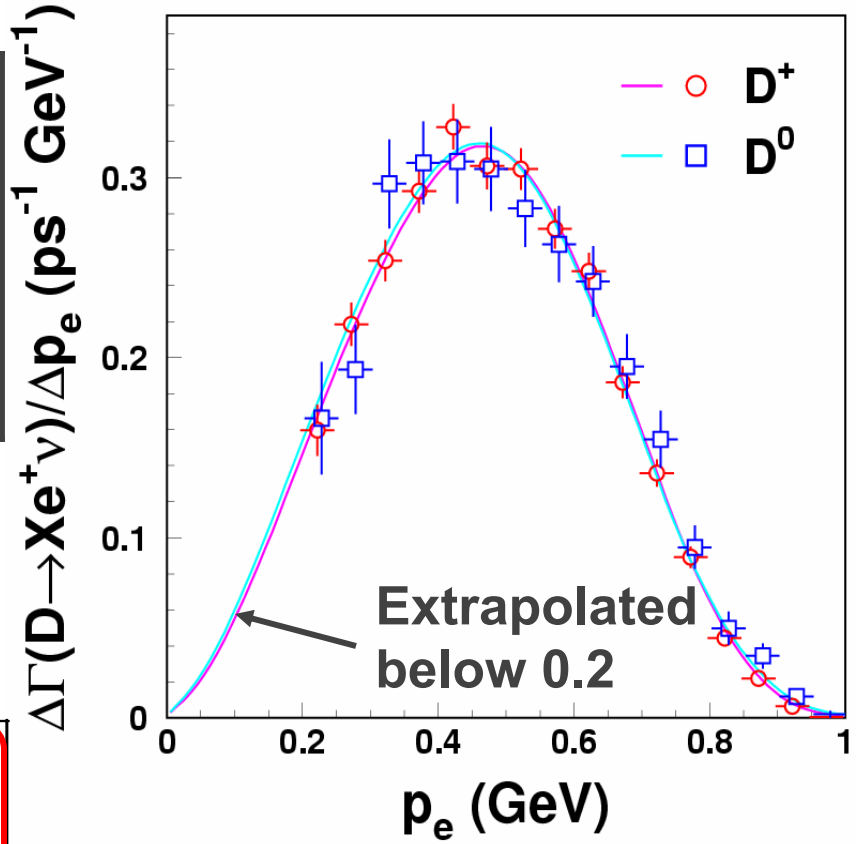
Consistent with the known exclusive modes saturating the inclusive branching fractions .

$$\frac{\Gamma_{D^+}^{SL}}{\Gamma_{D^0}^{SL}} = \frac{B_{D^+}^{SL}}{B_{D^0}^{SL}} \times \frac{\tau_{D^0}}{\tau_{D^+}} = 0.985 \pm 0.028 \pm 0.015$$

$$\Gamma(D^0 \rightarrow Xe^+\nu_e) = 0.1574 \pm 0.0041 \pm 0.0032 \text{ ps}^{-1}$$

$$\Gamma(D^+ \rightarrow Xe^+\nu_e) = 0.1551 \pm 0.0020 \pm 0.0031 \text{ ps}^{-1}$$

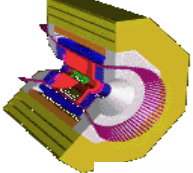
Consistent with isospin symmetry



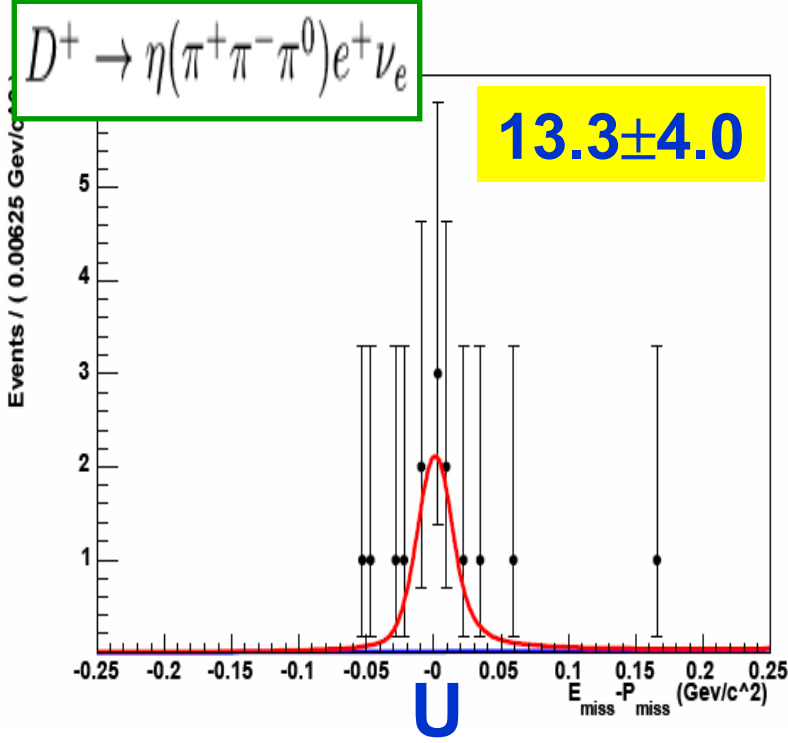
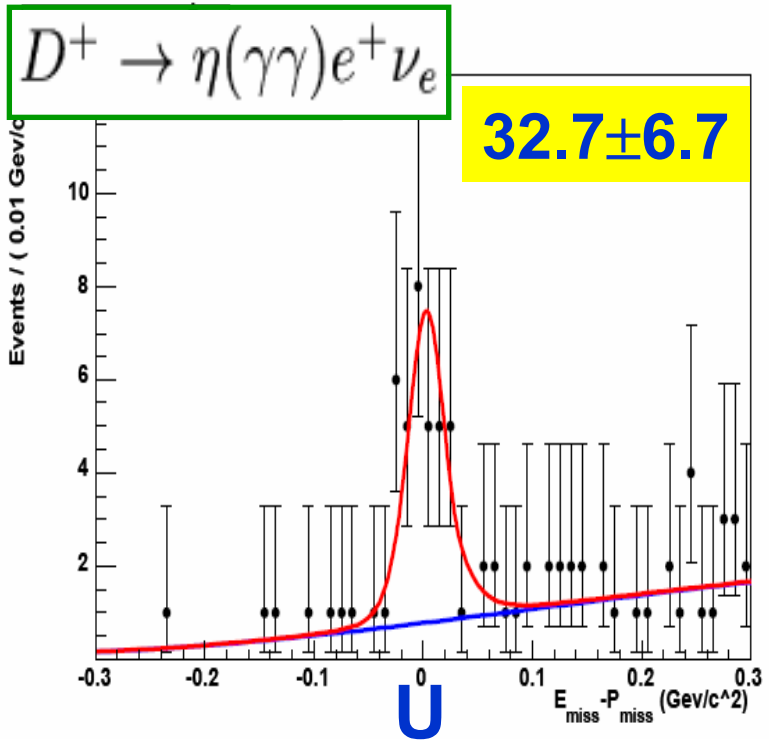
CLEO-c 281 pb<sup>-1</sup>

hep-ex/0604044

Submitted to PRL



# First observation of $D^+ \rightarrow \eta e^+ \nu$



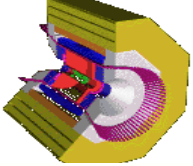
**Preliminary**

**CLEO-c 281 pb<sup>-1</sup> Preliminary**

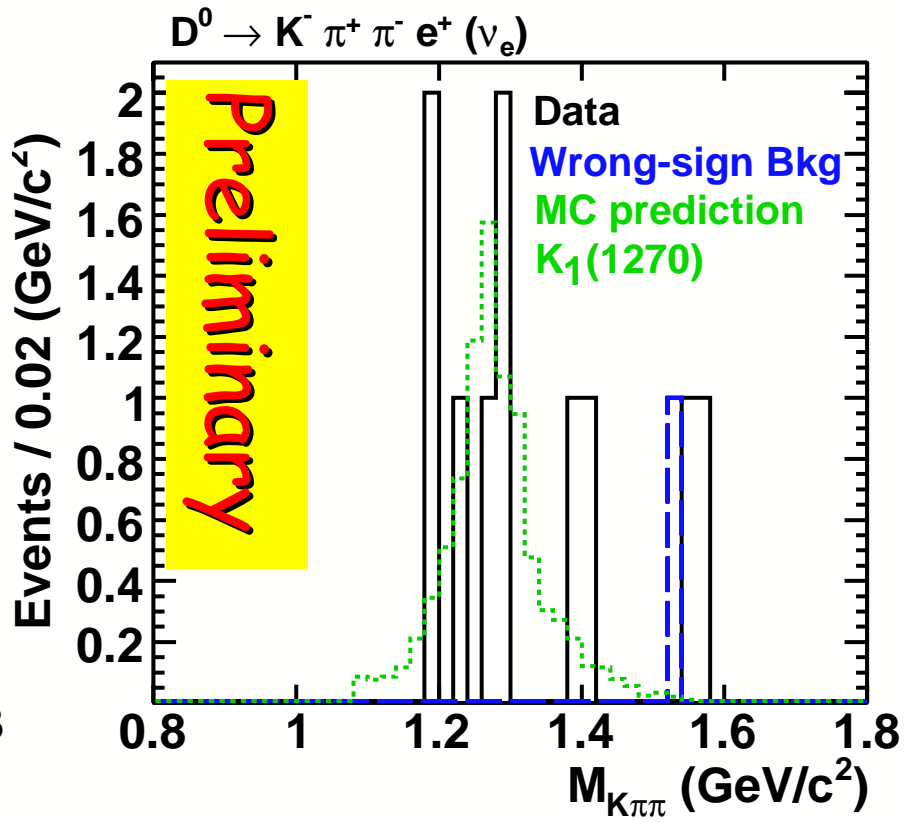
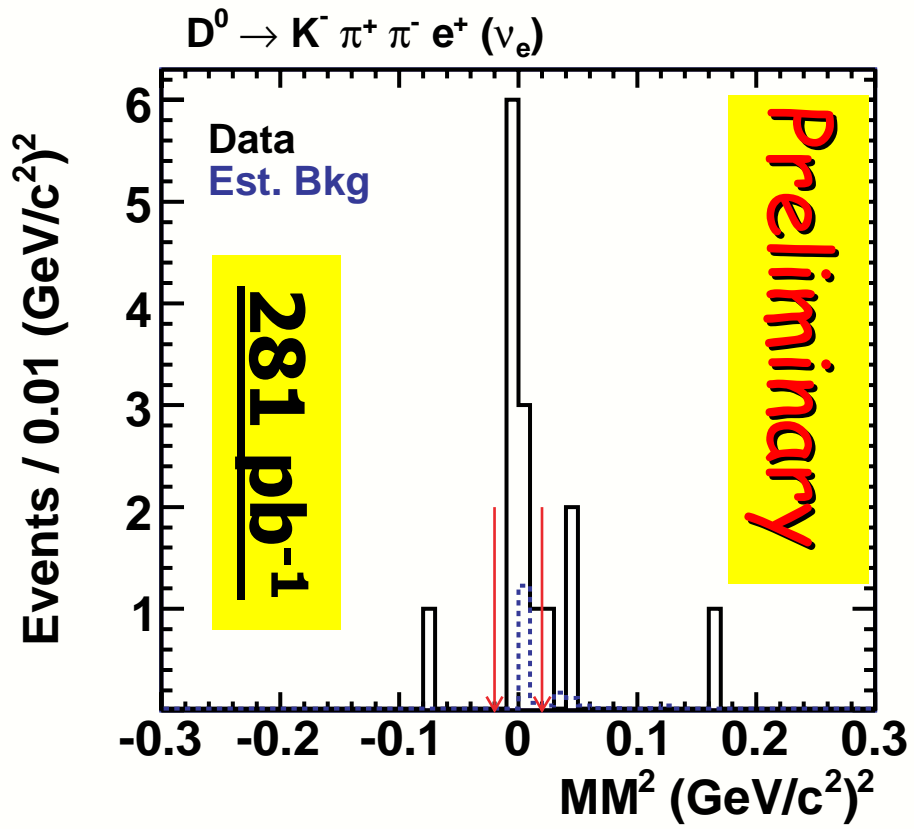
**PDG (2004)**

$B(D^+ \rightarrow \eta e^+ \nu) = (1.29 \pm 0.19 \pm 0.07) \times 10^{-3}$   
 $B(D^+ \rightarrow \eta' e^+ \nu)_{combined} < 3 \times 10^{-4}$  (90% C.L.)  
 $B(D^+ \rightarrow \phi e^+ \nu) < 2 \times 10^{-4}$  (90% C.L.)

$B(D^+ \rightarrow \eta l^+ \nu) < 5 \times 10^{-3}$  (90% C.L.)  
 $B(D^+ \rightarrow \eta' \mu^+ \nu) < 1.1\%$  (90% C.L.)  
 $B(D^+ \rightarrow \phi e^+ \nu) < 2.09\%$  (90% C.L.)



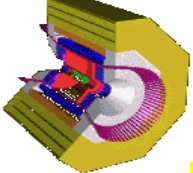
# Evidence for $D^0 \rightarrow K^- \pi^+ \pi^- e^+ \nu$



$$B(D^0 \rightarrow K^- \pi^+ \pi^- e^+ \nu) = (2.9^{+1.5}_{-1.1} \pm 0.5) \times 10^{-4}$$

$$B(D^0 \rightarrow K_1(1270) e^+ \nu) * B(K_1(1270) \rightarrow K^- \pi^+ \pi^-) = (2.2^{+1.4}_{-1.0} \pm 0.2) \times 10^{-4}$$

Consistent with ISGW2 prediction



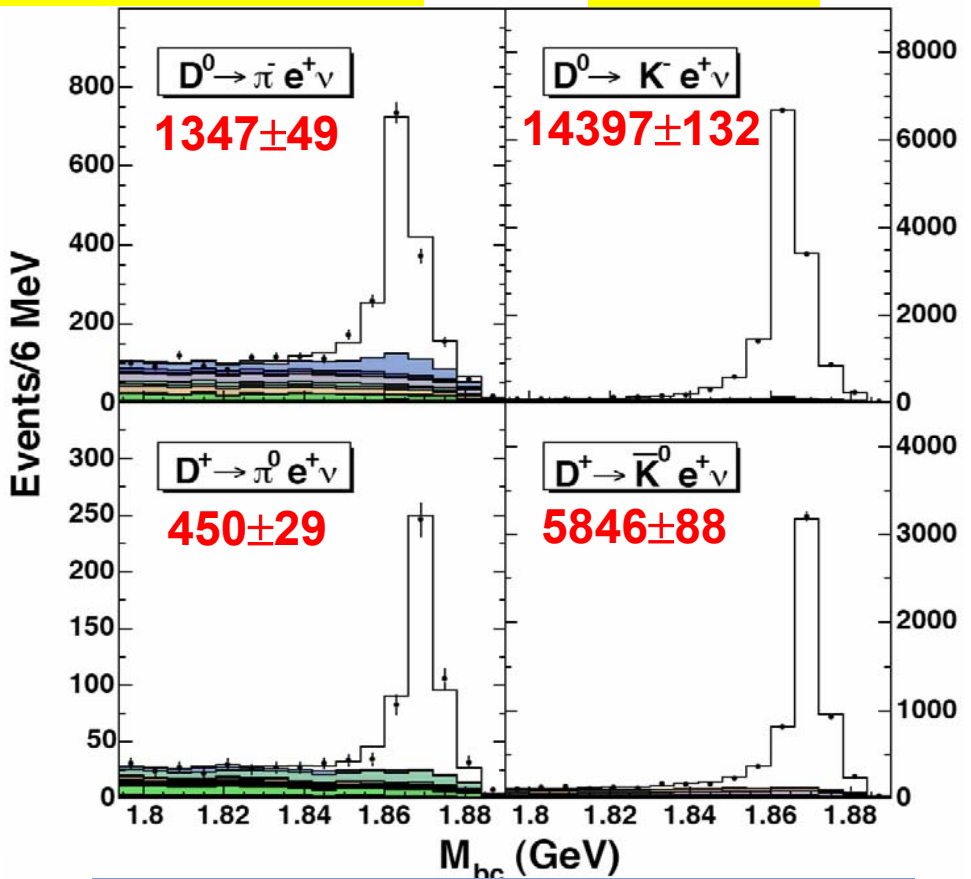
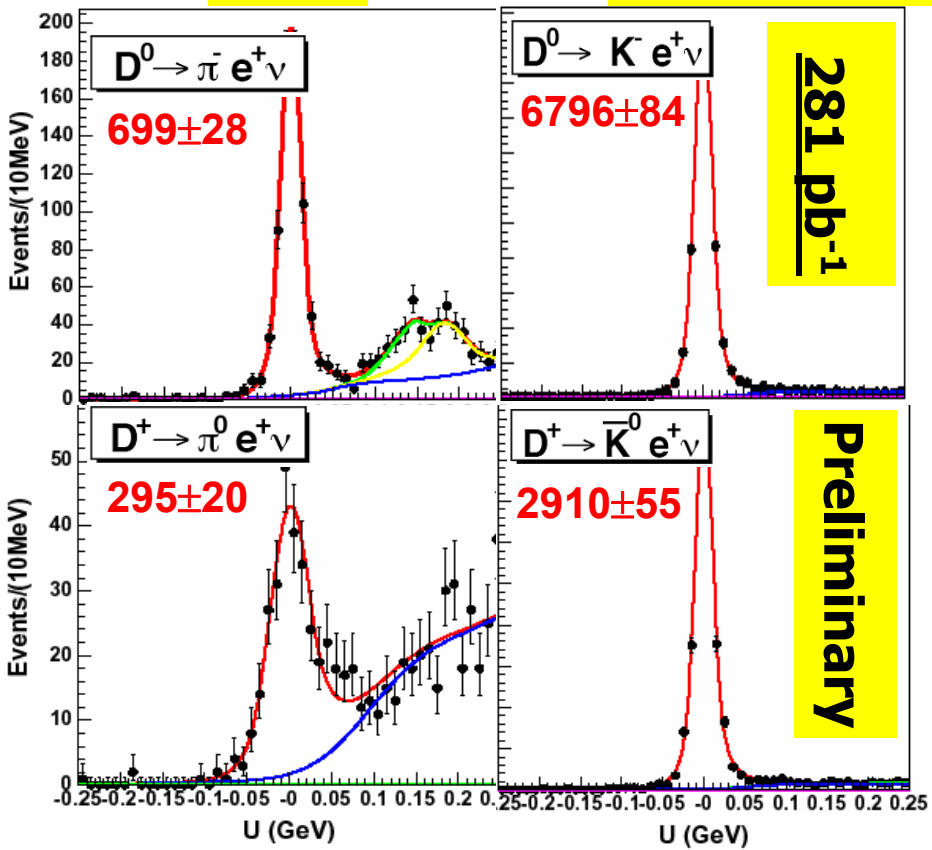
# D<sup>0</sup>/D<sup>+</sup> → K/π e ν Tag & Untag



**Tag**

**40% common samples!**

**Untag**

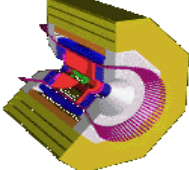


$$U = E_{\text{miss}} - |\mathbf{P}_{\text{miss}}| \text{ (GeV)}$$

$E_{\text{miss}}$  and  $\mathbf{P}_{\text{miss}}$  are missing energy and momentum of the event

$$\Delta E = E_{K(\pi)} + E_e + |\mathbf{p}_{\text{miss}}| - E_{\text{beam}}$$

$$M_{bc} = \sqrt{E_{\text{beam}}^2 - (\mathbf{p}_{K(\pi)} + \mathbf{p}_e + \zeta \mathbf{p}_{\text{miss}})^2}$$



# D → K/πeν BFs (Tag/Untag)



40% common samples, do NOT average them!

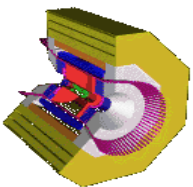
D Decay	Tag	Br. Frac. (%)	Untag	PDG (%)
$D^0 \rightarrow K^- e^+ \nu$	<b><math>3.58 \pm 0.05 \pm 0.05</math></b>		<b><math>3.56 \pm 0.03 \pm 0.11</math></b>	<b><math>3.62 \pm 0.16</math></b>
$D^0 \rightarrow \pi^- e^+ \nu$	<b><math>0.309 \pm 0.012 \pm 0.006</math></b>		<b><math>0.301 \pm 0.011 \pm 0.010</math></b>	<b><math>0.311 \pm 0.030</math></b>
$D^+ \rightarrow \bar{K}^0 e^+ \nu$	<b><math>8.86 \pm 0.17 \pm 0.20</math></b>		<b><math>8.75 \pm 0.13 \pm 0.30</math></b>	<b><math>7.2 \pm 0.8</math></b>
$D^+ \rightarrow \pi^0 e^+ \nu$	<b><math>0.397 \pm 0.027 \pm 0.028</math></b>		<b><math>0.383 \pm 0.025 \pm 0.016</math></b>	<b><math>0.38 \pm 0.19</math></b>

281 pb<sup>-1</sup>

Ratio	Measured (%)	PDG (%)	Ratio	Measured
$\frac{D^0 \rightarrow \pi^- e^+ \nu}{D^0 \rightarrow K^- e^+ \nu}$	<b><math>8.5 \pm 0.3 \pm 0.1</math></b>	<b><math>8.6 \pm 0.7</math></b>	$\frac{\Gamma(D^0 \rightarrow \pi^- e^+ \nu)}{\Gamma(D^+ \rightarrow \pi^0 e^+ \nu)}$	<b><math>1.95 \pm 0.15 \pm 0.14</math></b> <b><math>1.99 \pm 0.15 \pm 0.10</math></b>
$\frac{D^+ \rightarrow \pi^0 e^+ \nu}{D^+ \rightarrow \bar{K}^0 e^+ \nu}$	<b><math>4.4 \pm 0.3 \pm 0.1</math></b>	<b><math>4.6 \pm 1.4 \pm 1.7</math></b>	$\frac{\Gamma(D^0 \rightarrow K^- e^+ \nu)}{\Gamma(D^+ \rightarrow \bar{K}^0 e^+ \nu)}$	<b><math>1.02 \pm 0.02 \pm 0.02</math></b> <b><math>1.03 \pm 0.02 \pm 0.04</math></b>

Preliminary





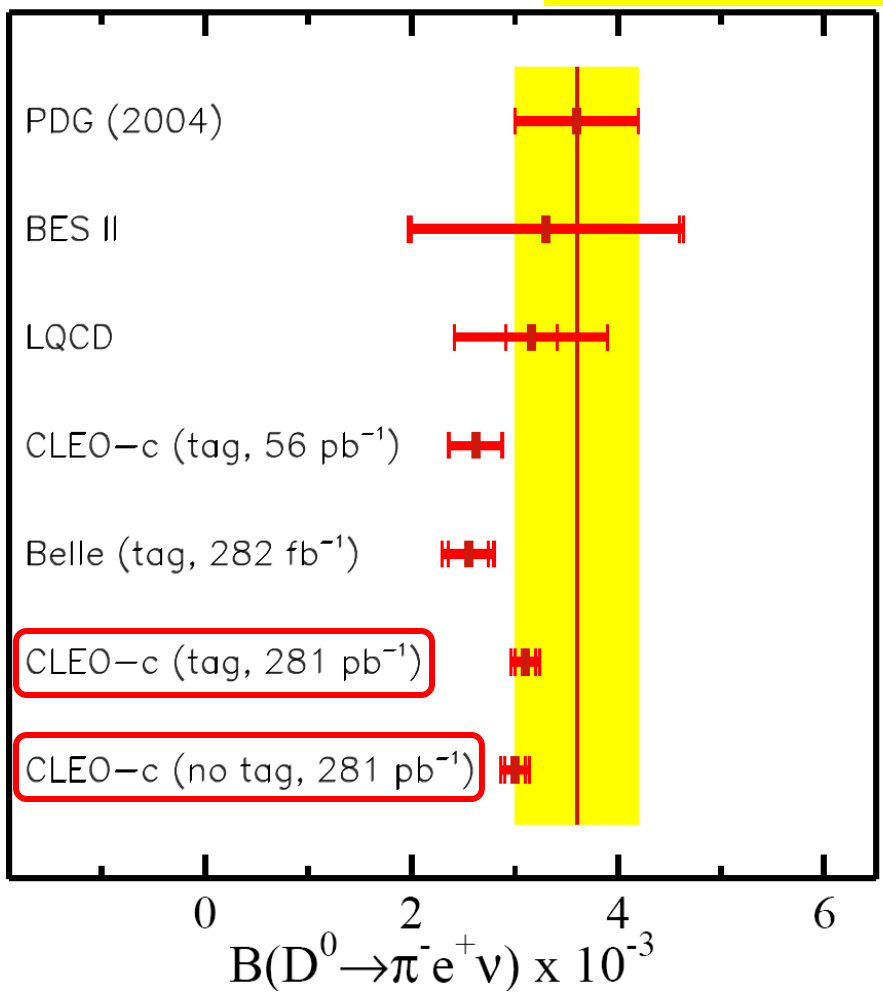
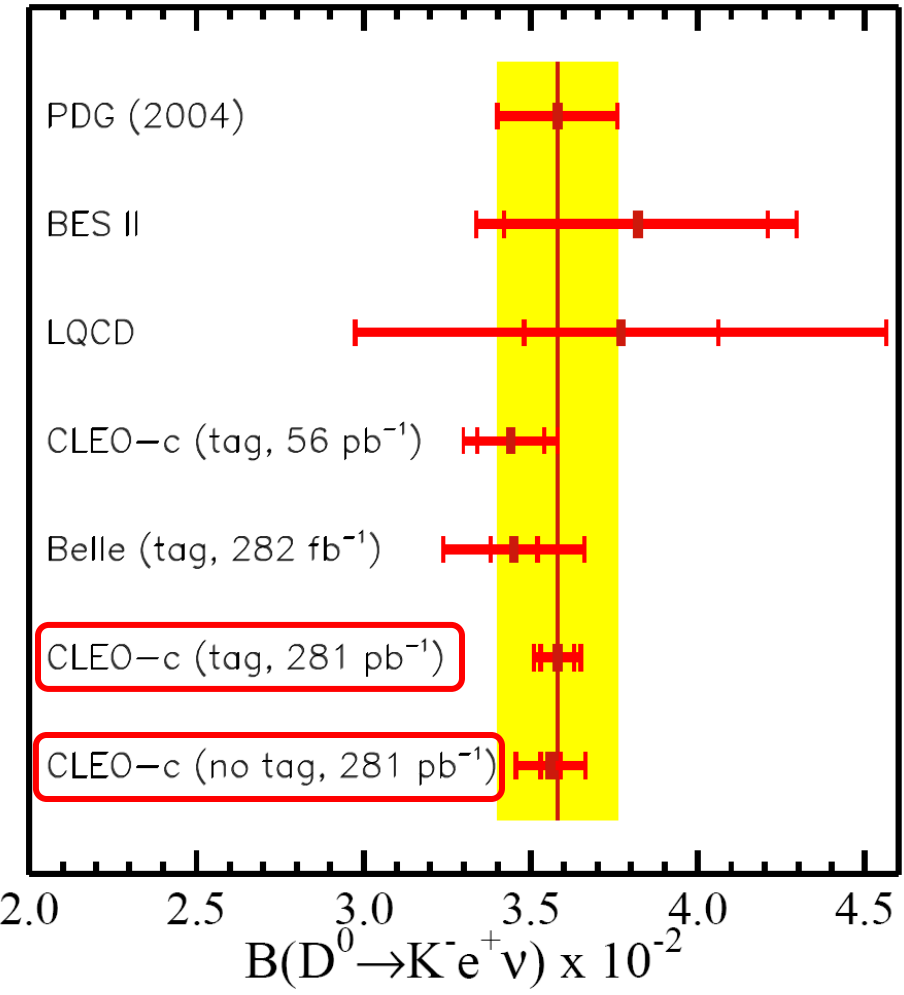
# Comparison with LQCD/other exp.

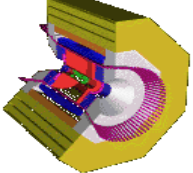


## CLEO-c 281 pb<sup>-1</sup> Preliminary

$D^0 \rightarrow K^- e^+ \nu$

$D^0 \rightarrow \pi^- e^+ \nu$

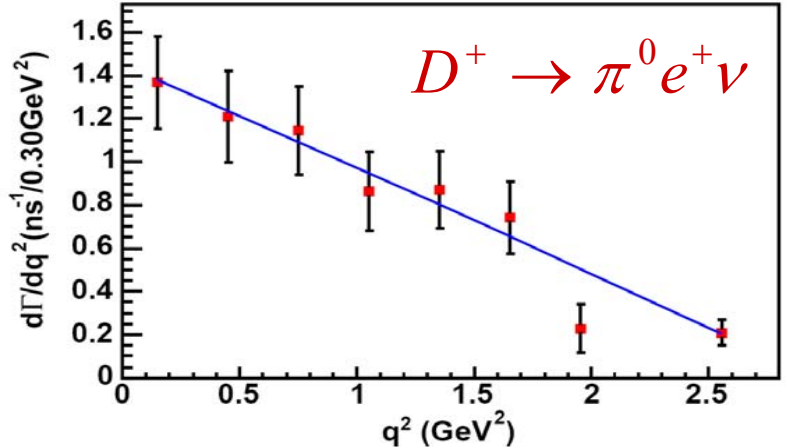
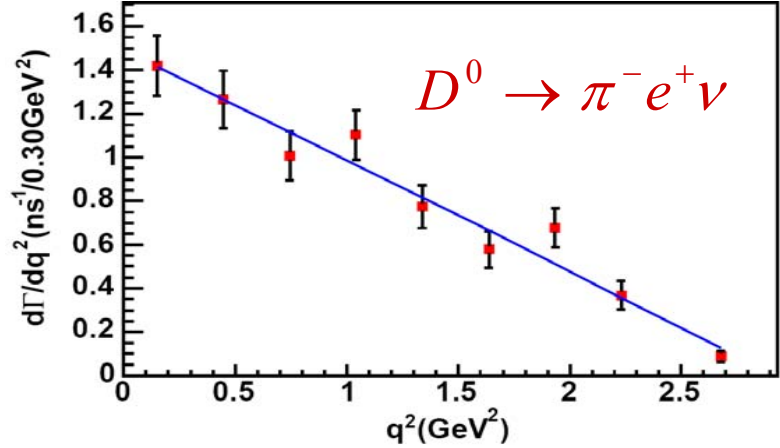
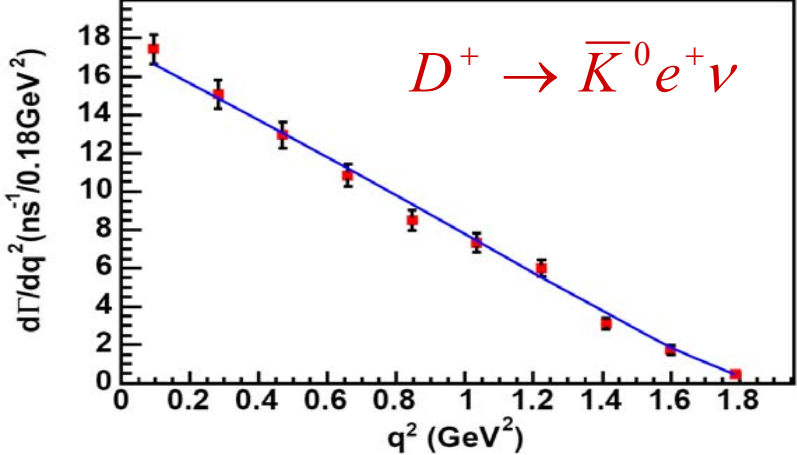
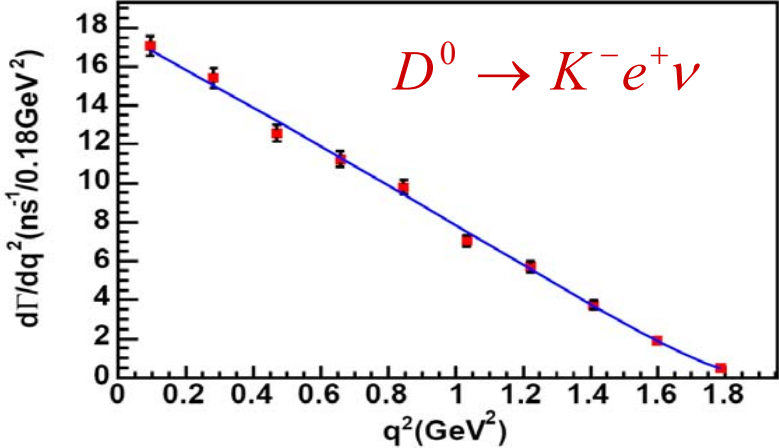




# Form Factor Fit (Tag)

Preliminary

281 pb<sup>-1</sup> at  $\Psi(3770)$



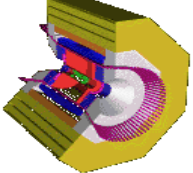
Simple Pole Model

$$f^+(q^2) = \frac{f^+(0)}{(1 - q^2/m_{pole}^2)}$$

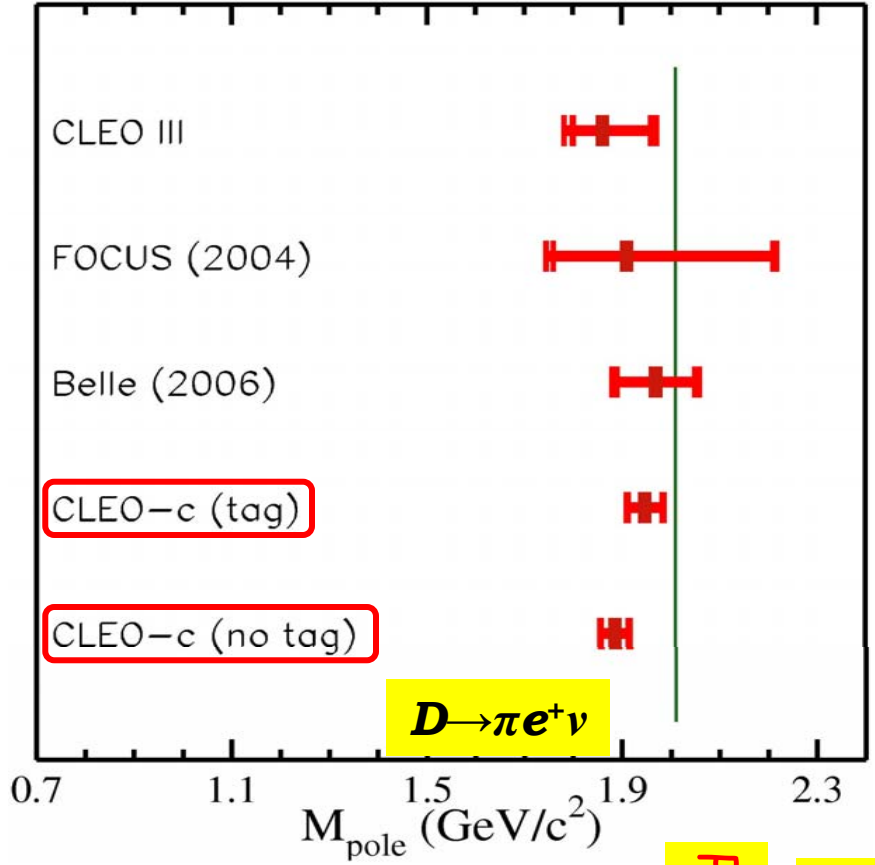
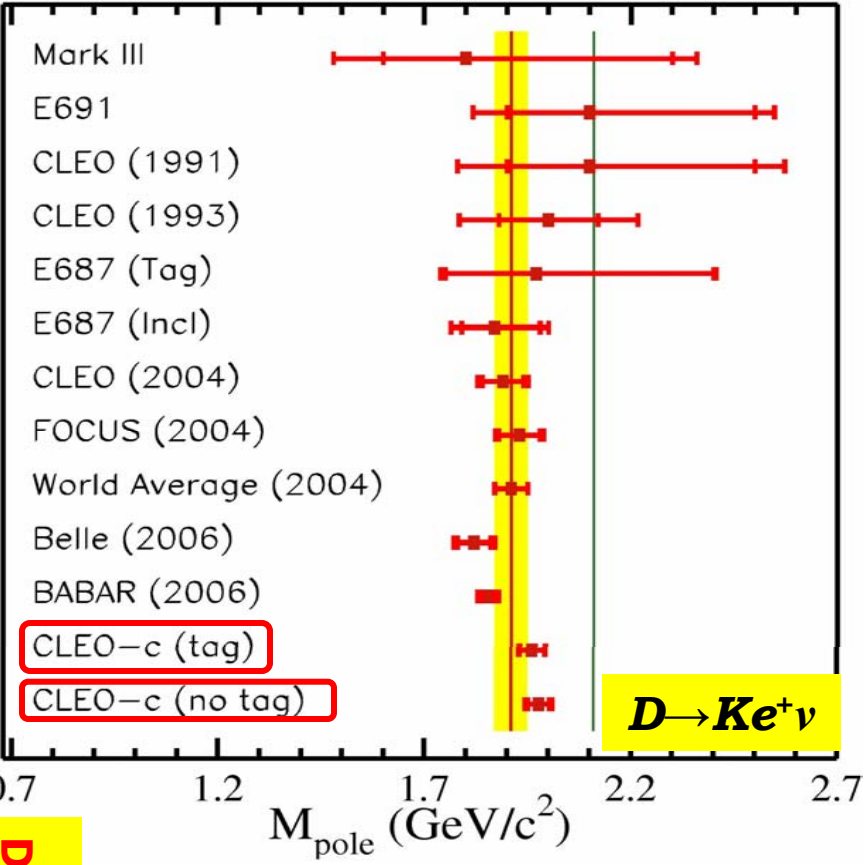
Modified Pole Model

$$f^+(q^2) = \frac{f^+(0)}{(1 - q^2/m_{pole}^2)(1 - \alpha q^2/m_{pole}^2)}$$

Hill series expansion (Phys. Lett. B 633, 61 (2006))



# Form Factors (Tag/Untag)

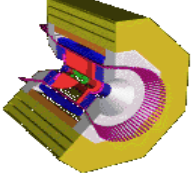


Don't average!

Decay Mode	Mpole (Tag)	Mpole (Untag)
$D \rightarrow Ke^+\nu$ (av. $D^0$ & $D^+$ )	$1.96 \pm 0.03 \pm 0.01$	$1.98 \pm 0.03 \pm 0.02$
$D \rightarrow \pi e^+\nu$ (av. $D^0$ & $D^+$ )	$1.95 \pm 0.04 \pm 0.02$	$1.88 \pm 0.03 \pm 0.02$

Preliminary

281 pb<sup>-1</sup>

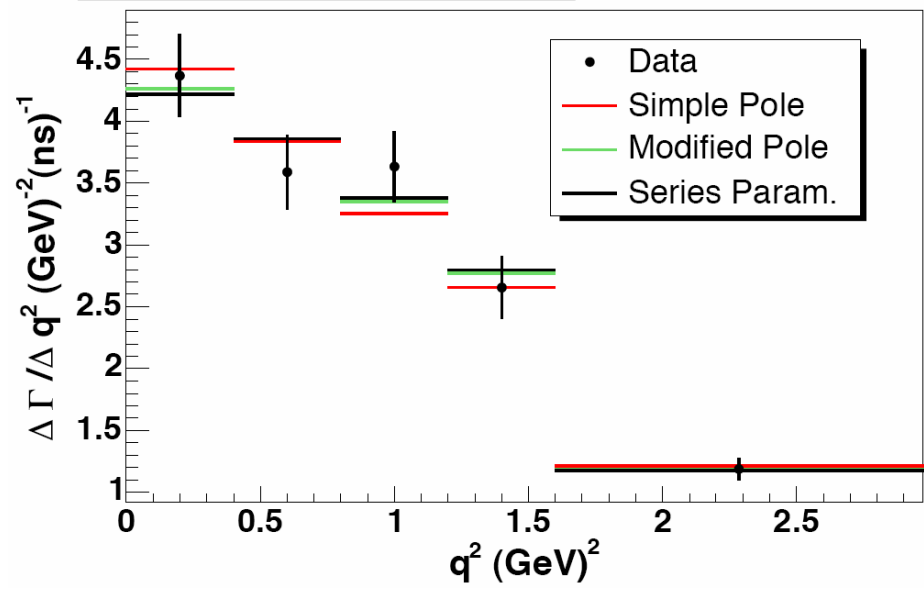


# Form Factor Fits (untag)

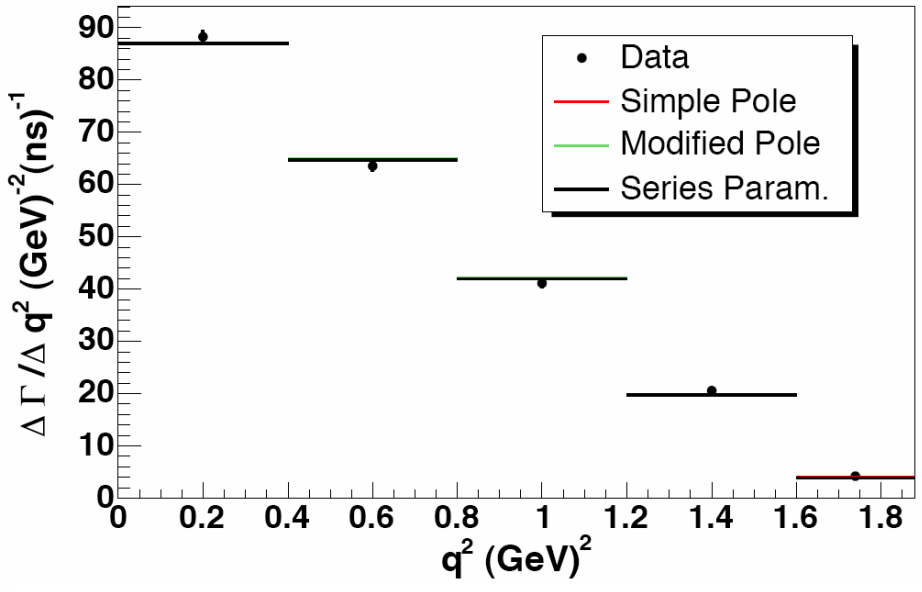


## CLEO-c 281 pb<sup>-1</sup> Preliminary Results:

D<sup>0</sup> → π e<sup>+</sup> ν Form Factor Fits

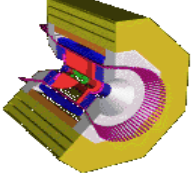


D<sup>0</sup> → K<sup>-</sup> e<sup>+</sup> ν Form Factor Fits



**To be submitted to PRL & PRD**

Decay Mode	V <sub>cx</sub>   ± (stat) ± (syst) ± (theory)	PDG Value
D → πeν (av. D <sup>0</sup> & D <sup>+</sup> )	<b>0.229 ± 0.007 ± 0.005 ± 0.024</b>	0.224 ± 0.012
D → Keν (av. D <sup>0</sup> & D <sup>+</sup> )	<b>0.996 ± 0.008 ± 0.015 ± 0.104</b>	0.976 ± 0.014

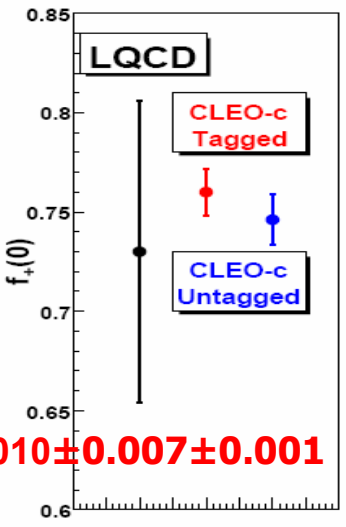
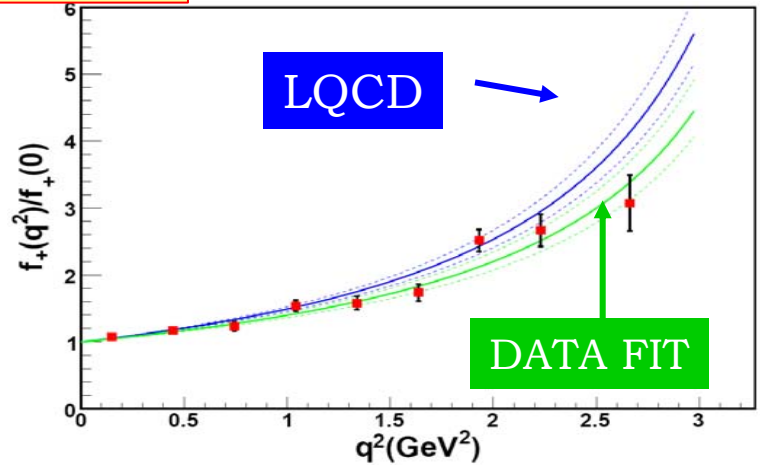
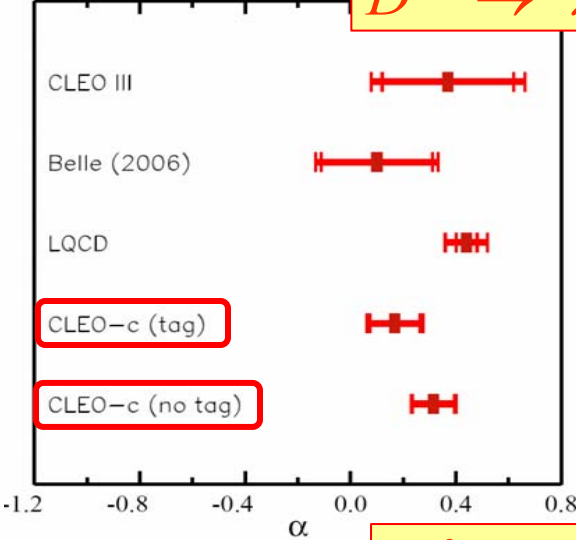
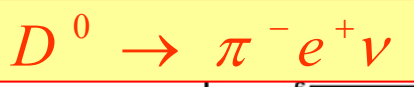
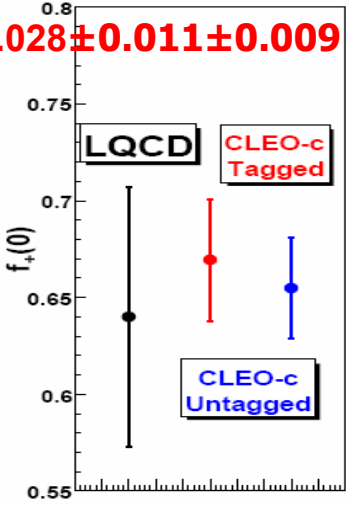


# Form Factors and Test of LQCD

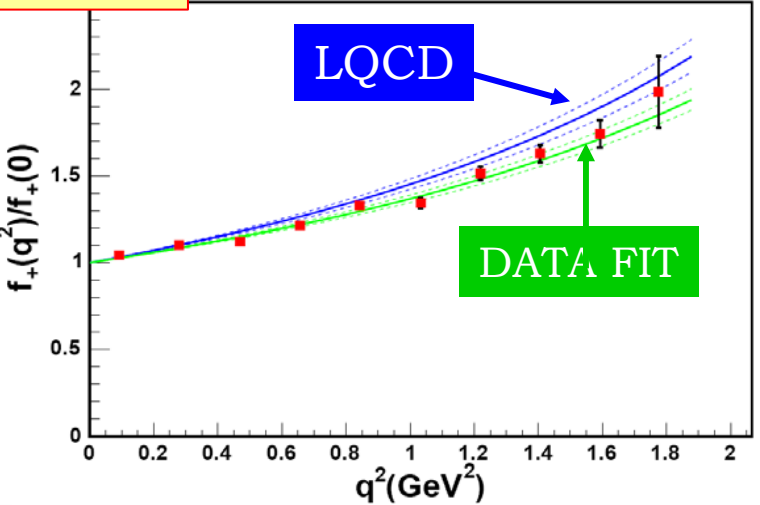
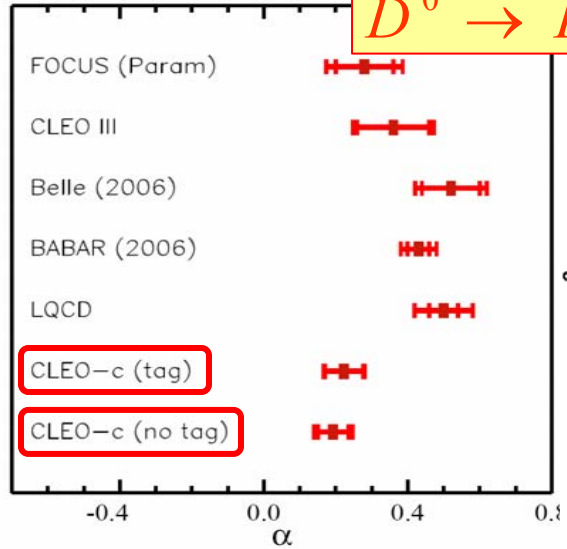


**281 pb<sup>-1</sup> Preliminary**

$$0.670 \pm 0.028 \pm 0.011 \pm 0.009$$



$$0.760 \pm 0.010 \pm 0.007 \pm 0.001$$







# $V_{cs}$ and $V_{cd}$ Results

Combine  $|V_{cx}|f_+(0)$  values from fits with unquenched LQCD results for  $f_+(0)$   
(Phys. Rev. Lett. 94, 011601 (2005)) to extract  $|V_{cs}|$  and  $|V_{cd}|$ .

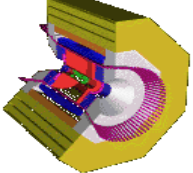
Decay Mode	$ V_{cx}  \pm (\text{stat}) \pm (\text{syst}) \pm (\text{theory})$	PDG Value
$D \rightarrow \pi e \nu$ (tag)	$0.234 \pm 0.010 \pm 0.004 \pm 0.024$	
$D \rightarrow \pi e \nu$ (untag)	$0.229 \pm 0.007 \pm 0.005 \pm 0.024$	$0.224 \pm 0.012$
$D \rightarrow K e \nu$ (tag)	$1.014 \pm 0.013 \pm 0.009 \pm 0.106$	
$D \rightarrow K e \nu$ (untag)	$0.996 \pm 0.008 \pm 0.015 \pm 0.104$	$0.976 \pm 0.014$

Preliminary

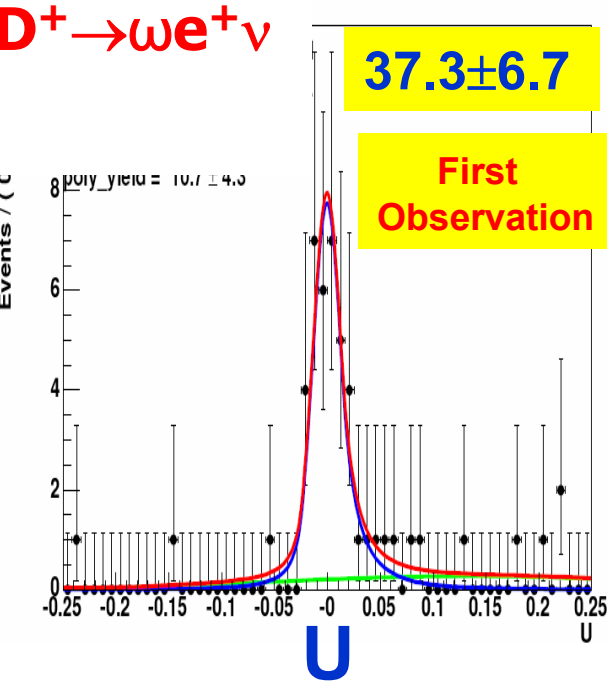
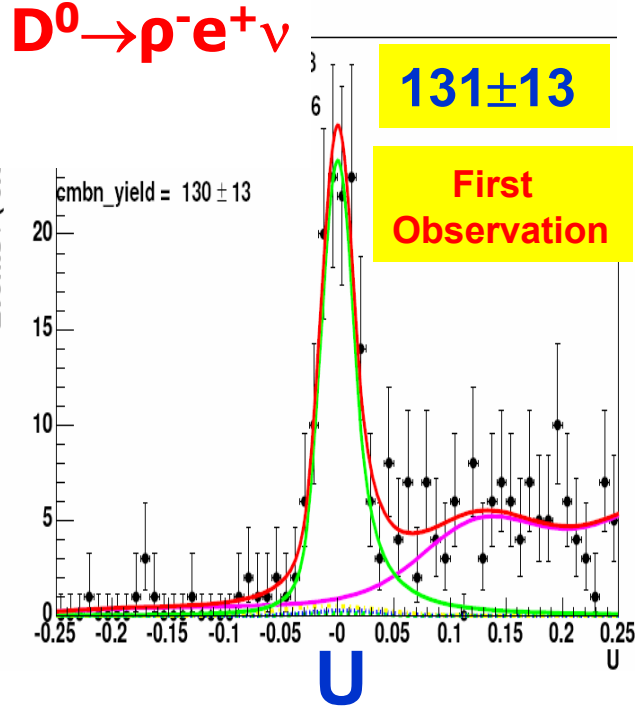
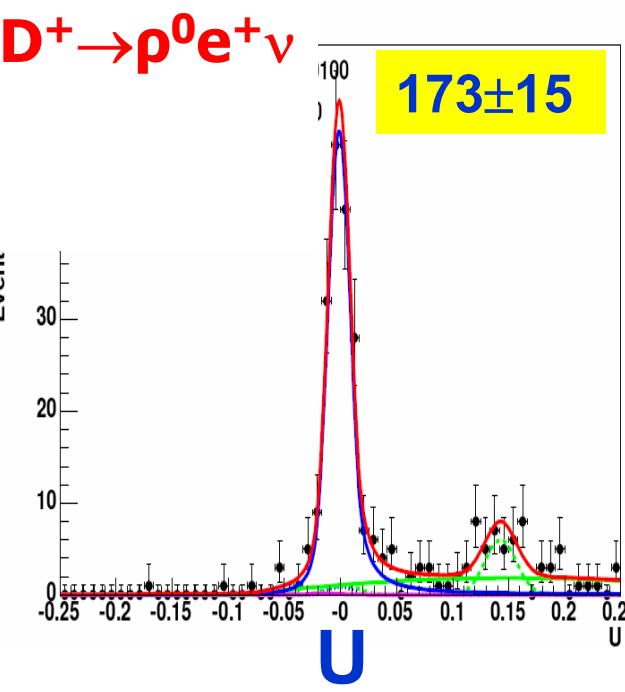
Tag/Untag: 40% of comment sample. **DO NOT AVERAGE!!!**

Expt. uncertainties  $V_{cs} < 2\%$   $V_{cd} \sim 4\%$  LQCD uncertainty 10%

Since  $V_{cs}$  ( $W \rightarrow cs$  LEP) and  $V_{cd}$  ( $\nu N$ ) are well measured, good agreement between PDG and CLEO-c results is primarily a check of the LQCD value for  $f_+(0)$ . Nevertheless, the most precise & robust  $V_{cs}$  &  $V_{cd}$  determinations using semileptonic decays to date.



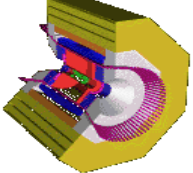
# Exclusive $D \rightarrow \rho e \nu$ Results



## CLEO-c 281 pb<sup>-1</sup> Preliminary Results:

$B(D^0 \rightarrow \rho^- e^+ \nu) = (0.156 \pm 0.016 \pm 0.009)\%$   
 $B(D^+ \rightarrow \rho^0 e^+ \nu) = (0.232 \pm 0.020 \pm 0.012)\%$   
 $B(D^+ \rightarrow \omega e^+ \nu) = (0.149 \pm 0.027 \pm 0.005)\%$

$$\frac{\Gamma(D^0 \rightarrow \rho^- e \nu)}{2 \cdot \Gamma(D^+ \rightarrow \rho^0 e \nu)} = 0.85 \pm 0.11$$



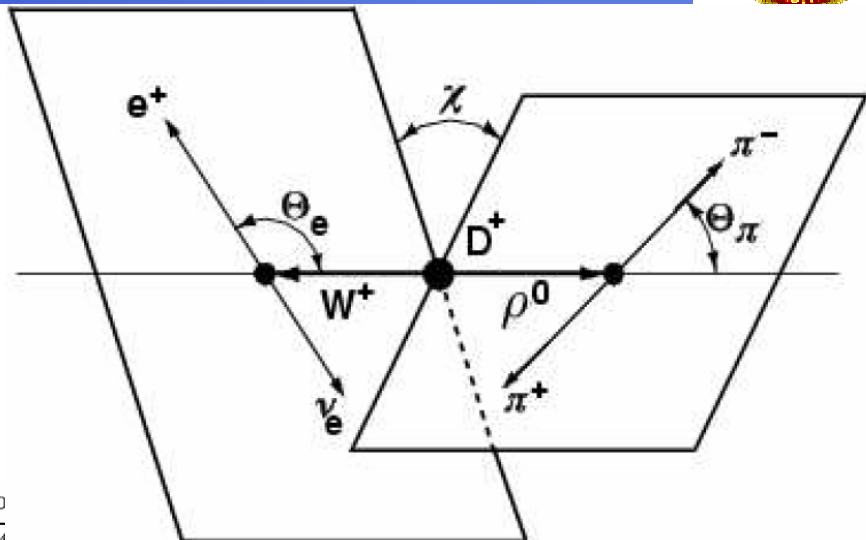
# kinematics & Decay Rate



Five kinematic variables describe the decay rate:

$$q^2, \cos \theta_e, \cos \theta_\pi, \chi, m(\pi\pi)$$

We fit to the decay rate

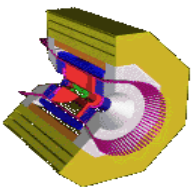


$$\frac{d\Gamma}{dq^2 d \cos \theta_\pi d \cos \theta_e d \chi} = \mathcal{B}(\rho^0 \rightarrow \pi\pi) \frac{3G_F^2}{8(4\pi)^4} |V_{cs}|^2 \frac{p_{\rho^0}}{M_D} \{ (1 + \cos \theta_e)^2 \sin^2 \theta_\pi |H_+(q^2)|^2 + (1 - \cos \theta_e)^2 \sin^2 \theta_\pi |H_-(q^2)|^2 + 4 \sin^2 \theta_e \cos^2 \theta_\pi |H_0(q^2)|^2 + 4 \sin \theta_e (1 + \cos \theta_e) \sin \theta_\pi \cos \theta_\pi \cos \chi H_+(q^2) H_0(q^2) - 4 \sin \theta_e (1 - \cos \theta_e) \sin \theta_\pi \cos \theta_\pi \cos \chi H_-(q^2) H_0(q^2) - 2 \sin^2 \theta_e \sin^2 \theta_\pi \cos 2\chi H_+(q^2) H_-(q^2) \}$$

Dependence on the form factors enters through  $H_+$ ,  $H_-$  and  $H_0$ .

$$A_{1(2)}(q^2) = \frac{A_{1(2)}(0)}{1 - q^2/M_A^2}; \quad V(q^2) = \frac{V(0)}{1 - q^2/M_V^2}$$

$$R_V \equiv \frac{V(0)}{A_1(0)}; \quad R_2 \equiv \frac{A_2(0)}{A_1(0)}$$



# Exclusive $D \rightarrow \rho e \nu$ Results



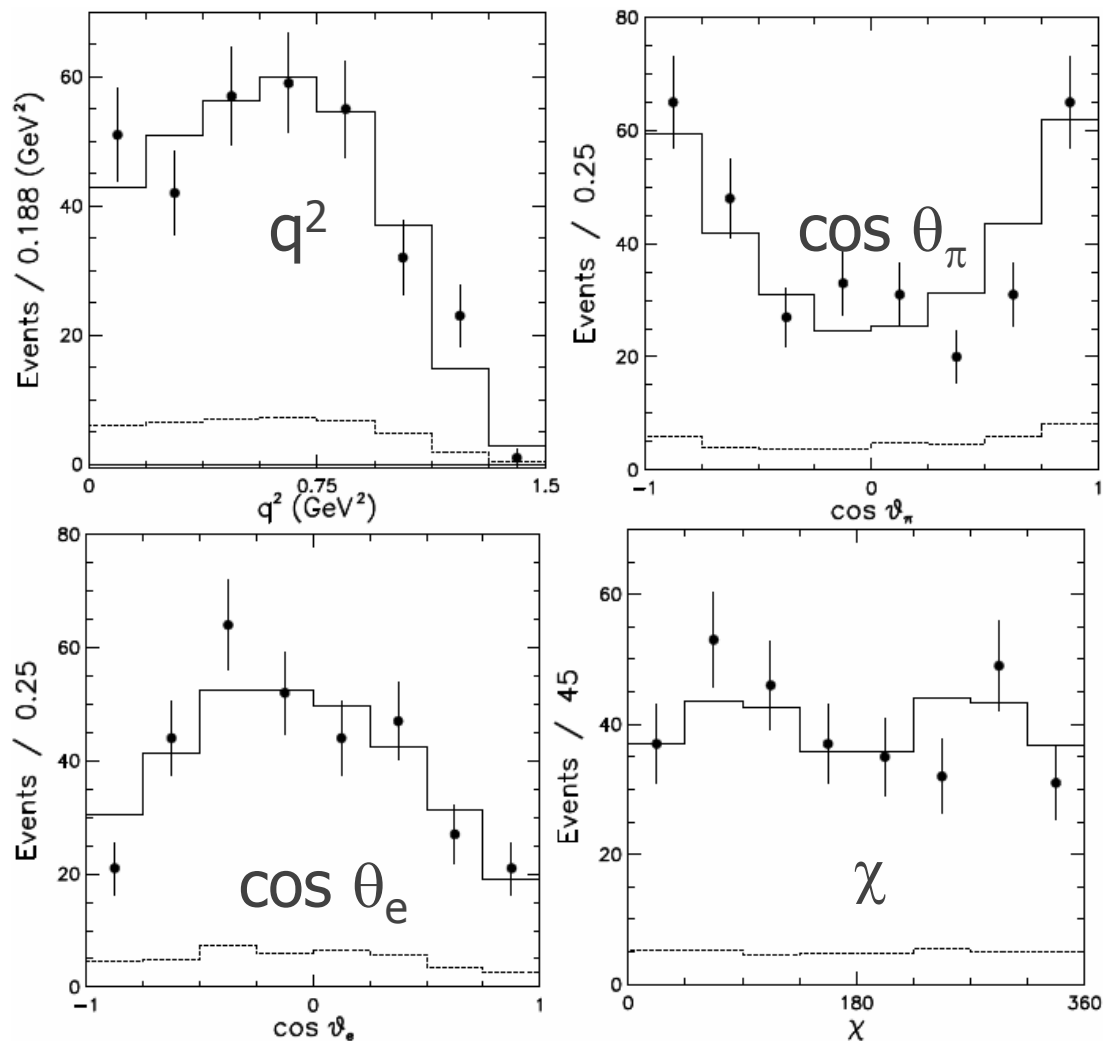
**Preliminary  
Results:**

**CLEO-c 281 pb<sup>-1</sup>:**

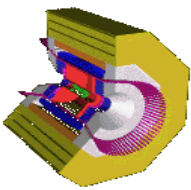
$$R_V = 1.40 \pm 0.25 \pm 0.03$$

$$R_2 = 0.57 \pm 0.18 \pm 0.06$$

**First measurement**



Line is projection for fitted  $R_V, R_2$



# Exclusive Semileptonic $b \rightarrow u$



Important for  $V_{ub}$ , but challenges:

- Large  $b \rightarrow c$  backgrounds
- Missing neutrino

$$B \rightarrow X_u \ell \nu \quad X_u = \pi^\pm, \pi^0, \eta, \rho^\pm, \rho^0, \omega, \eta' \quad \ell = \mu, e$$

Neutrino Reconstruction:  $p_\nu = p_{\text{beam}} - p_{\text{visible}}$

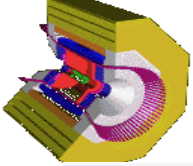
**Signature: peaks in**

$$\Delta E = (E_{X_u} + E_\ell + E_\nu) - E_{\text{beam}}$$
$$M_{X_u \ell \nu} = \sqrt{E_{\text{beam}}^2 - |\vec{p}_{X_u} + \vec{p}_\ell + \vec{p}_\nu|^2}$$

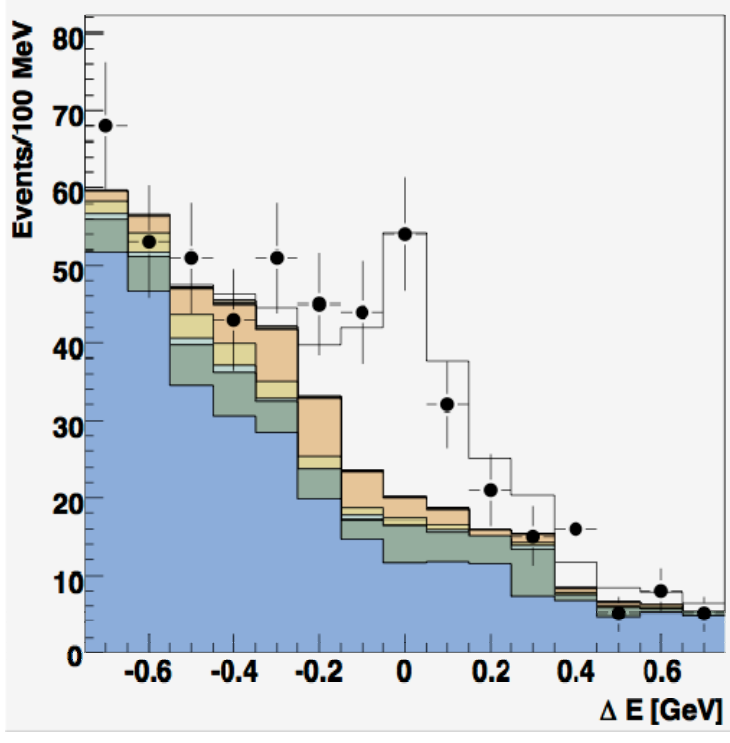
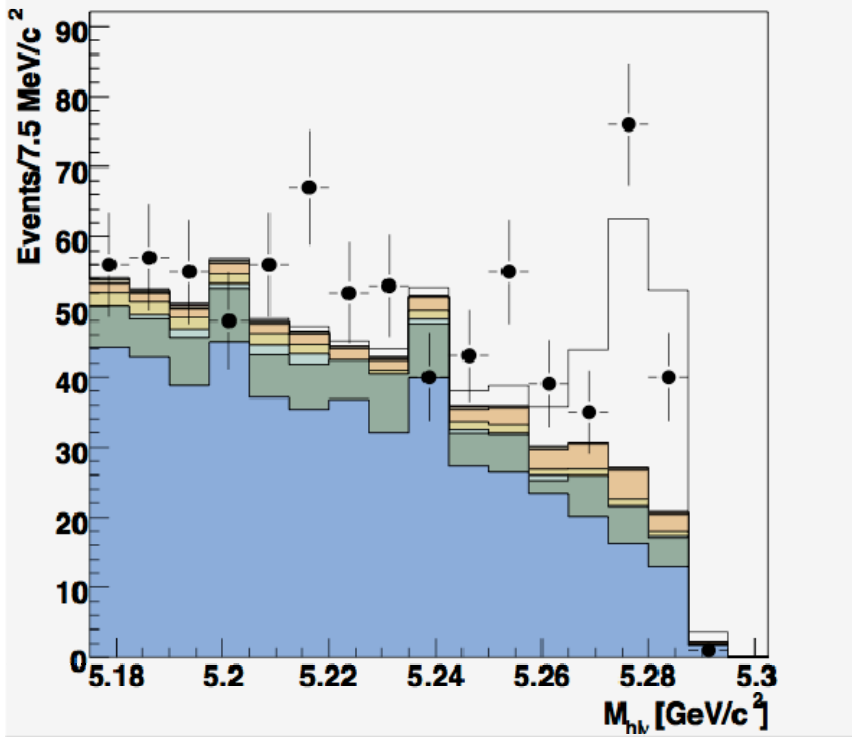
**CLEO data at Y(4S): CLEO II, II.V and III**

- $15.8 \times 10^6$  BB events (60% more data)
- Supersedes PRD68, 072003 (lower lepton Pt)





# Exclusive Semileptonic $B \rightarrow \pi l \nu$



**Preliminary**

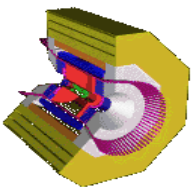
**15.8x10<sup>6</sup> BB evts**

**CLEO II, II.V, III**

*CLEO Preliminary*

$q^2$ Bin [ $\text{GeV}^2$ ]	Branching Fraction [ $\times 10^{-4}$ ] (errors: stat., syst., theo.)
$q^2 < 2$	$0.13 \pm 0.07 \pm 0.02 \pm 0.00$
$2 < q^2 < 8$	$0.27 \pm 0.08 \pm 0.03 \pm 0.00$
$8 < q^2 < 16$	$0.56 \pm 0.09 \pm 0.06 \pm 0.01$
$q^2 > 16$	$0.40 \pm 0.08 \pm 0.05 \pm 0.02$
<b>Total</b>	<b><math>1.37 \pm 0.16 \pm 0.13 \pm 0.02</math></b>

**Supersedes PRD68, 072003**



# Exclusive Semileptonic $B \rightarrow \rho l \nu$



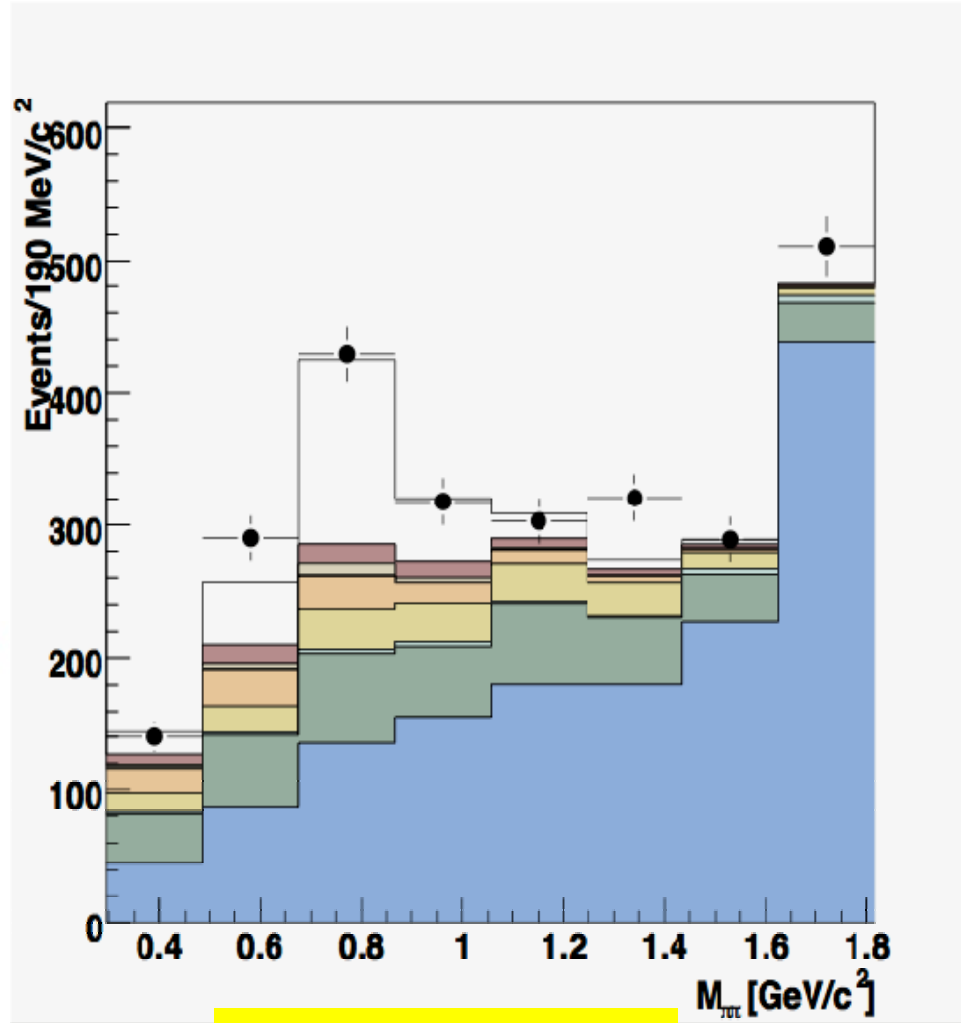
## Preliminary Results:

CLEO Y(4S) data

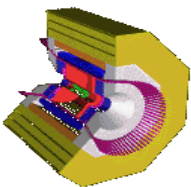
$15.8 \times 10^6$  BB evts

CLEO Preliminary

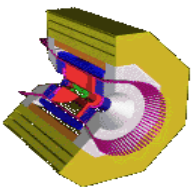
$q^2$ Range [GeV <sup>2</sup> ]	$\cos \Theta_{wl}$ Range	Branching Fraction [ $\times 10^{-4}$ ] (errors: stat., syst., theo.)
$q^2 < 2$	$-1 < \cos \Theta_{wl} < 1$	$0.45 \pm 0.20 \pm 0.15 \pm 0.02$
$2 < q^2 < 8$	$-1 < \cos \Theta_{wl} < 1$	$0.95 \pm 0.20 \pm 0.30 \pm 0.07$
$8 < q^2 < 16$	$0 < \cos \Theta_{wl} < 1$	$0.75 \pm 0.16 \pm 0.12 \pm 0.01$
$q^2 > 16$	$0 < \cos \Theta_{wl} < 1$	$0.35 \pm 0.07 \pm 0.04 \pm 0.01$
$q^2 > 8$	$-1 < \cos \Theta_{wl} < 0$	$0.43 \pm 0.18 \pm 0.30 \pm 0.04$
<b>Total</b>		<b><math>2.91 \pm 0.38 \pm 0.37 \pm 0.07</math></b>



**Mass of  $\pi\pi$**



**back-up slides**



# Exclusive $D^+ \rightarrow K^- \pi^+ e^+ \nu$



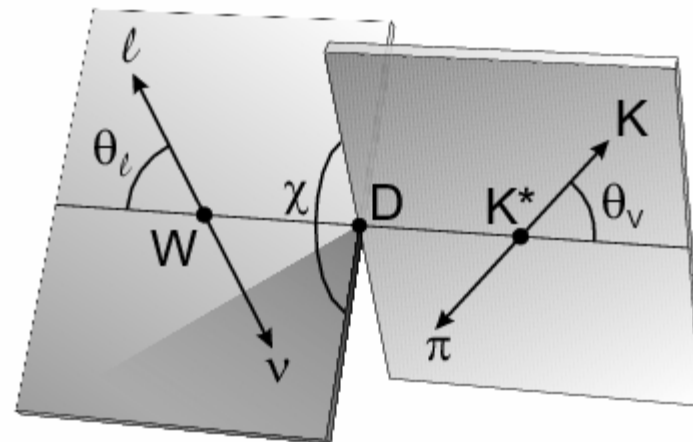
**hep-ex/0606010**

$K^- \pi^+$  mostly  $K^*$  with some  
s-wave (first seen by FOCUS)

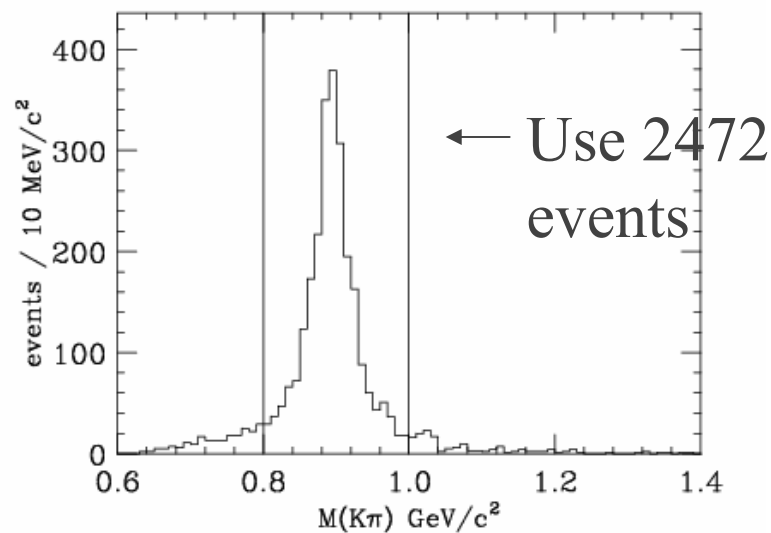
For  $D \rightarrow V e^+ \nu$ , use 3 helicity  
amplitudes  $H_0(q^2)$ ,  $H_+(q^2)$ ,  
&  $H_-(q^2)$

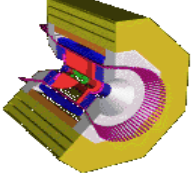
Add  $h_0(q^2) \bullet H_0(q^2)$  to account for  
s-wave term

Use  $281 \text{ pb}^{-1}$  tagged analysis



Submitted to PRD

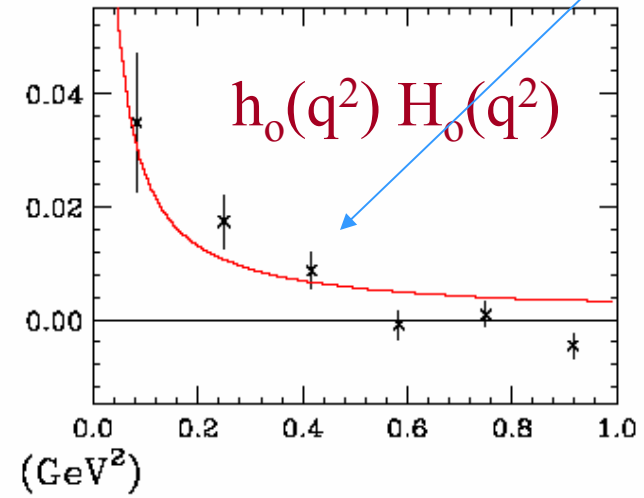
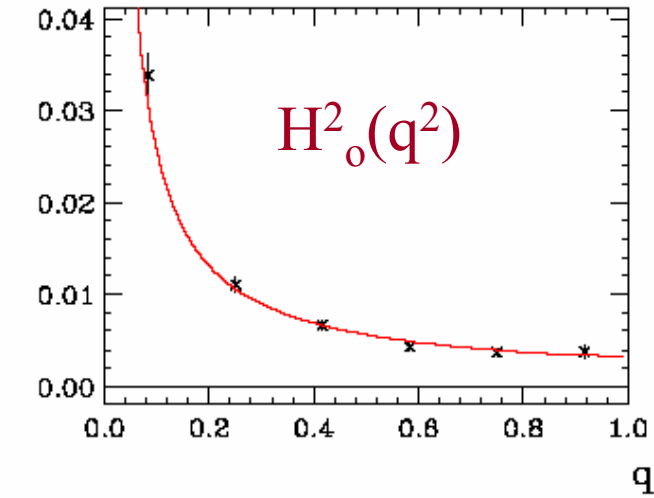
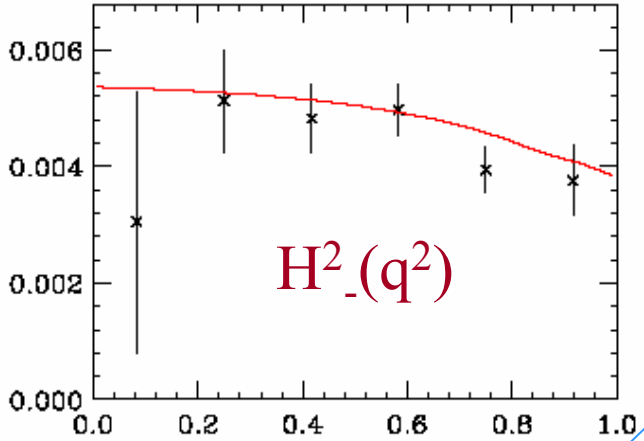
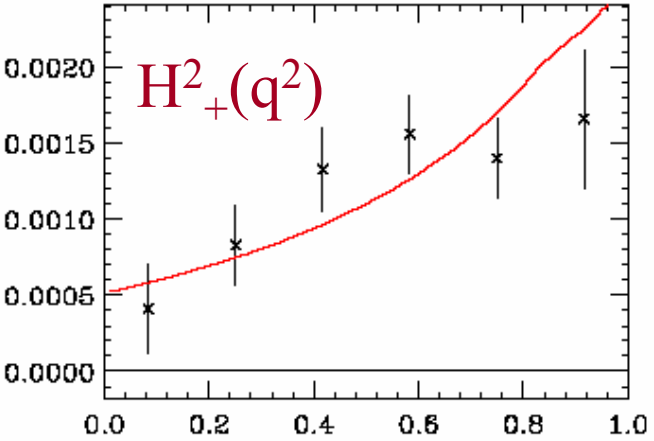




# Exclusive $D^+ \rightarrow K^- \pi^+ e^+ \nu$



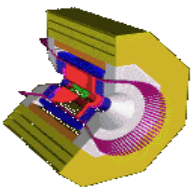
**hep-ex/0606010**



Significant s-wave amplitude confirmed

No evidence for d or f wave

Submitted to PRD



# Neutrino Reconstruction

## *Neutrino Reconstruction*

*We use the whole event to reconstruct the event missing four-momentum ( $P_{miss}$ ). This can be associated with a neutrino if the missing mass is consistent with zero.*

$$P_{miss} = P_{event} - \sum P_{charged} - \sum P_{neutral}$$

$$\Delta E = E_{K(\pi)} + E_e + |\mathbf{p}_{miss}| - E_{beam}$$

$$M_{bc} = \sqrt{E_{beam}^2 - (\mathbf{p}_{K(\pi)} + \mathbf{p}_e + \beta \mathbf{p}_{miss})^2}$$

*$\beta$  is a correction to the missing momentum*

$$E_{K(\pi)} + E_e + \beta |\mathbf{p}_{miss}| - E_{beam} \equiv 0$$



# Form Factors

$$f_+(q^2) = \frac{f_+(0)}{1 - \alpha} \frac{1}{1 - q^2/m_{pole}^2} + \frac{1}{\pi} \int_{(M_D+m)^2}^{\infty} dq'^2 \frac{\text{Im}(f(q'^2))}{q'^2 - q^2}$$

*General dispersion relation -- too complicated...*

$$f_+(q^2) = \frac{f_+(0)}{\left(1 - q^2/m_{pole}^2\right)}$$

*Simple Pole Model*

$$f_+(q^2) = \frac{f_+(0)}{\left(1 - q^2/m_{pole}^2\right)\left(1 - \alpha q^2/m_{pole}^2\right)}$$

*Modified Pole Model*

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a_k(t_0) [z(q^2, t_0)]^k$$

*Series Parameterization*

$$t_{\pm} \equiv \left(M_D \pm m_{\pi(K)}\right)^2, \quad z(q^2, t_0) = \frac{\sqrt{t_{+} - q^2} - \sqrt{t_{+} - t_0}}{\sqrt{t_{+} - q^2} + \sqrt{t_{+} - t_0}}$$

*Hill & Becher, Phys. Lett. B 633, 61 (2006)*