1. Define a relation \( R \) on \( \mathbb{Z} \) by \( x R y \) if \( x \cdot y \geq 0 \). Prove or disprove the following:
   
   (a) \( R \) is reflexive;
   (b) \( R \) is symmetric;
   (c) \( R \) is transitive.

   **Solution:**

2. Let \( A = \{1, 2, 3, 4\} \). Give an example of a relation on \( A \) that is:
   
   (a) reflexive and symmetric, but not transitive;
   (b) symmetric and transitive, but not reflexive;
   (c) symmetric, but neither transitive nor reflexive.

   **Solution:**

3. Let \( R \) be an equivalence relation on \( A = \{a, b, c, d, e, f, g\} \) such that \( a R c, c R d, d R g, \) and \( b R f \). If there are three distinct equivalence classes that result from \( R \), then determine these equivalence classes and determine all elements of \( R \).

   **Solution:**

4. Define a relation \( R \) on \( \mathbb{Z} \) as \( x R y \) if and only if \( x^2 + y^2 \) is even. Prove \( R \) is an equivalence relation and determine its distinct equivalence classes.

   **Solution:**

5. Prove or disprove. If \( R \) and \( S \) are two equivalence relations on a set \( A \), then \( R \cap S \) is also an equivalence relation on \( A \).

   **Solution:**

6. Describe the partition of \( \mathbb{Z} \) resulting from the equivalence relation \( \equiv \ (\text{mod } 3) \).

   **Solution:**
7. Write the addition and multiplication tables for \( \mathbb{Z}_8 \).

Solution:

8. Prove or disprove. If \([a], [b] \in \mathbb{Z}_6\) and \([a][b] = [0]\), then either \([a] = [0]\) or \([b] = [0]\).

Solution: