Math 111, Fall 2014 - Homework # 3
Due Thursday, September 18, 2014, by 4:30 p.m.

Remember that you are required to fully explain all of your solutions.

1. For a real number $r$, define $A_r = \{r^2\}$, $B_r$ as the closed interval $[r - 1, r + 1]$, and $C_r$ as the interval $(r, \infty)$. For $S = \{1, 2, 4\}$, determine

   (a) $\bigcup_{r \in S} A_r$ and $\bigcap_{r \in S} A_r$

   (b) $\bigcup_{r \in S} B_r$ and $\bigcap_{r \in S} B_r$

   (c) $\bigcup_{r \in S} C_r$ and $\bigcap_{r \in S} C_r$.

Solution:

2. For $r \in \mathbb{R}^+$ ($\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$), let $A_r = \{x \in \mathbb{R} : |x| < r\}$. Determine $\bigcup_{r \in \mathbb{R}^+} A_r$ and $\bigcap_{r \in \mathbb{R}^+} A_r$.

Solution:

3. Determine which of the following are statements. For statements, determine if they are true or false.

   (i) Every even integer is a real number.

   (ii) $\mathbb{N} \notin P(\mathbb{N})$.

   (iii) The integer $x$ is divisible by 5.

   (iv) $\emptyset = \{\emptyset\}$.

Solution:

4. Express each statement or open sentence in one of the forms $P \land Q$, $P \lor Q$, or $\sim P$. Make sure to state exactly what statements $P$ and $Q$ stand for.

   (i) The matrix $A$ is not invertible.

   (ii) $x < y$

   (iii) At least one of the numbers $x$ and $y$ equals 0.

   (iv) $x \in A \cap B$

Solution:
5. State the negation of each of the following statements without using the word “not.”

(a) The real number \( r \) is at most 2.
(b) The absolute value of the number \( a \) is less than 3.
(c) Two sides of the triangle have the same length.
(d) No one expected it to rain.
(e) It is surprising that two students received the same exam score.

Solution:

6. Consider the statements \( P \): 17 is even and \( Q \): 19 is prime. Write each of the following statements in words and indicate whether it is true or false.

(a) \( \sim P \)
(b) \( P \land Q \)
(c) \( P \lor Q \)

Solution:

7. Without changing their meanings, convert each of the following sentences into a sentence having the form “If \( P \), then \( Q \).”

(a) Whenever three sides of a triangle are equal, the angles of the triangle are equal.
(b) The square of every integer is positive.
(c) The integer \( n^3 \) is even only if \( n \) is even.

Solution:

8. Without changing their meanings, convert each of the following sentences into a sentence having the form “\( P \) if and only if \( Q \).”

(a) If a function has constant derivative, it is linear, and conversely.
(b) For a circle to have both a perimeter and an area of \( 4\pi \), it is necessary and sufficient that its radius be 2.

Solution:

9. Consider the statements \( P \): \( \sqrt{2} \) is rational and \( Q \): \( \frac{22}{7} \) is rational. Write each of the following statements in words and indicate whether it is true or false.

(a) \( P \implies Q \)
(b) \( Q \implies P \)
(c) \( P \iff Q \)

Solution: